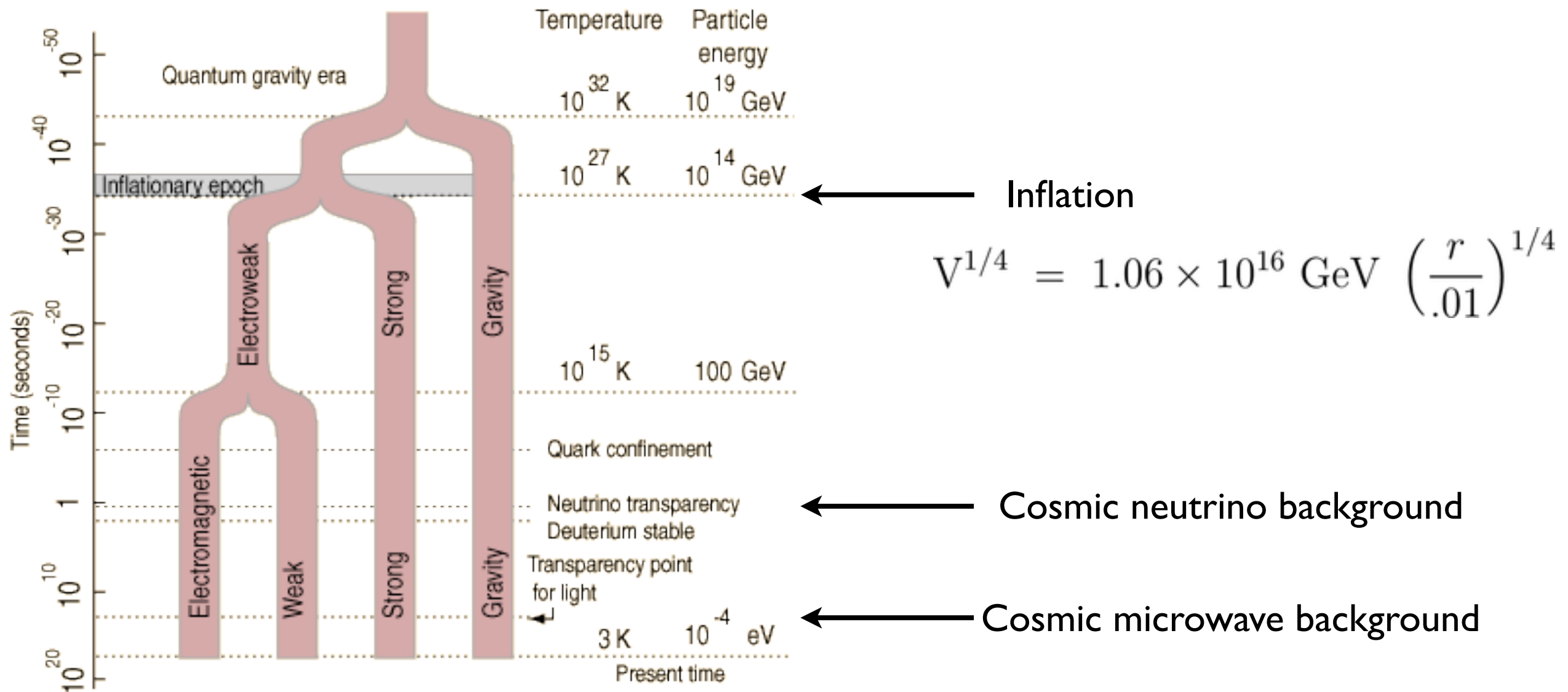# Cosmic Microwave Background
and
CMB-S4

CMB-S4: a coherent project building on CMB stage II & III projects
including participation by:
   the ACT, BICEP/KECK, SPT and Polarbear CMB teams;
   Argonne, FNAL, LBNL and SLAC national labs;
   and the HEP community.

John Carlstrom
for the CF5 Inflation and Neutrino topical groups
and the CMB-S4 collaboration

Early universe as a HEP lab

Temperature   Particle energy

Quantum gravity era 10^{32} K 10^{19} GeV

10^{27} K 10^{14} GeV

Inflationary epoch

Inflation

$$V^{1/4} = 1.06 \times 10^{16} \text{ GeV} \left(\frac{r}{.01}\right)^{1/4}$$

Electroweak   Strong   Gravity

10^{15} K   100 GeV

Electromagnetic   Weak   Strong   Gravity

Quark confinement

Neutrino transparency
Deuterium stable

Cosmic neutrino background

Transparency point for light

3 K 10^{-4} eV

Cosmic microwave background

Present time

Time (seconds)
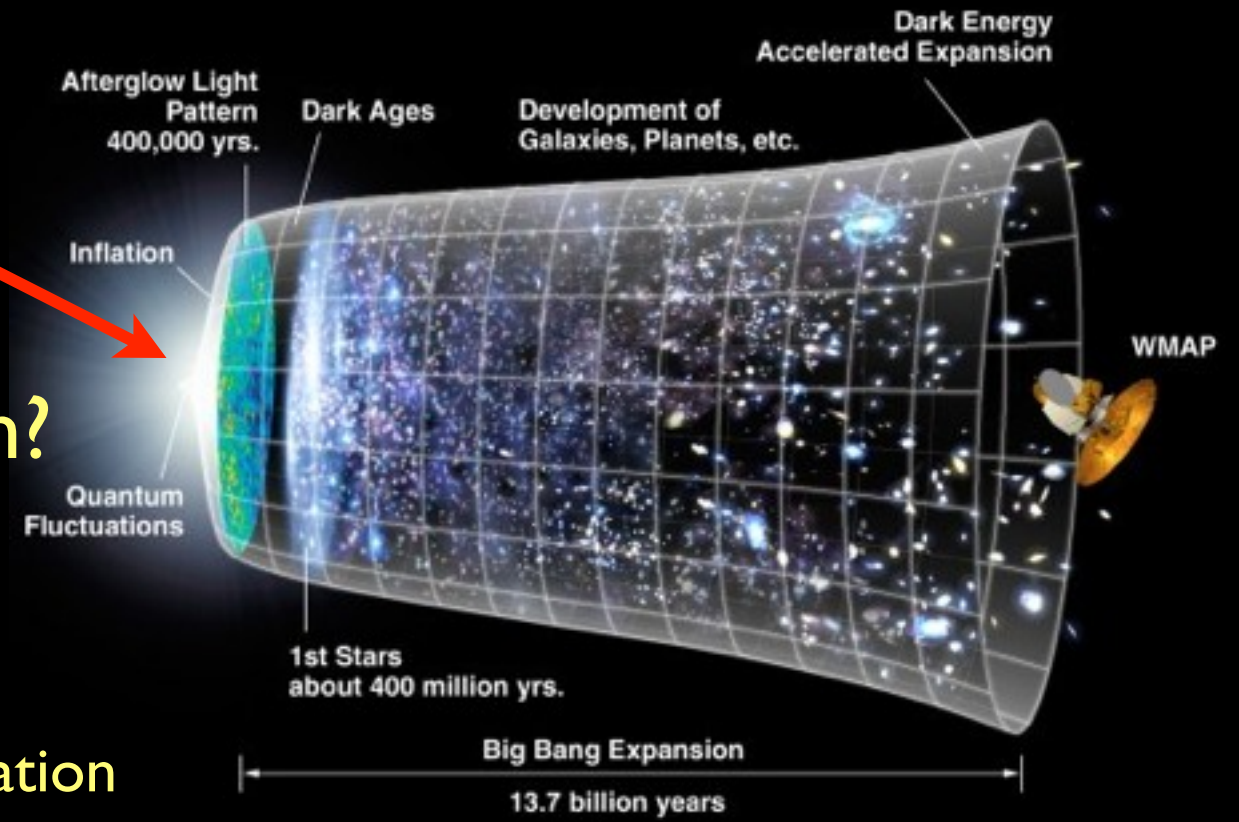10^{-50} 10^{-40} 10^{-30} 10^{-20} 10^{-10} 1 10^{10} 10^{20}

Inflation?

Universe expands by $>e^{60}$
solving smoothness problem,
flatness and more..

What drove inflation?
What is the energy scale of inflation?

- spectral index of fluctuations, n_s
- constrain tensor to scalar fluctuations
- inflationary gravitational wave B-mode polarization
- non-Gaussianity?

Afterglow Light
Pattern
400,000 yrs.

Dark Ages

Development of
Galaxies, Planets, etc.

Dark Energy
Accelerated Expansion

Inflation

WMAP

Quantum
Fluctuations

1st Stars
about 400 million yrs.

Big Bang Expansion
13.7 billion years

graphic from NASA/WMAP

Neutrinos?

N_{eff}

Effective number of relativistic species
'dark radiation' impacts intrinsic CMB power spectrum
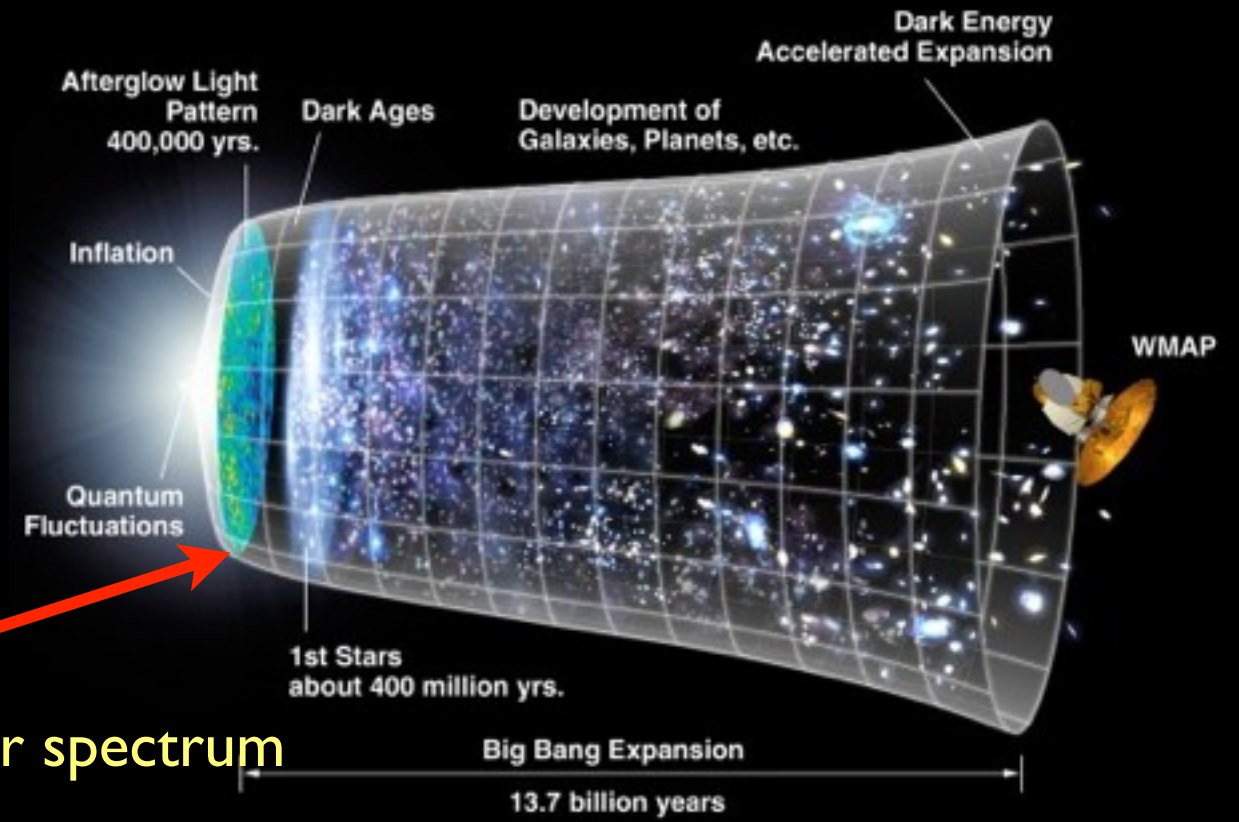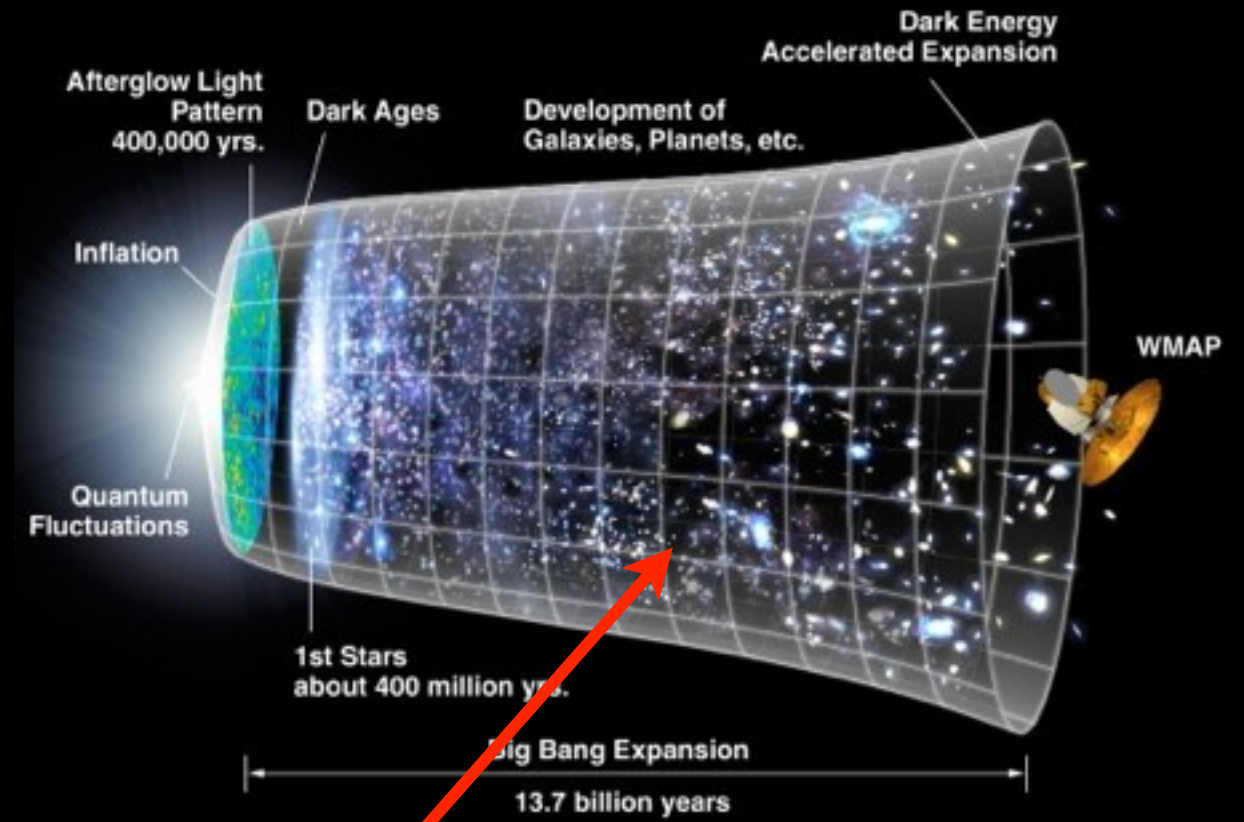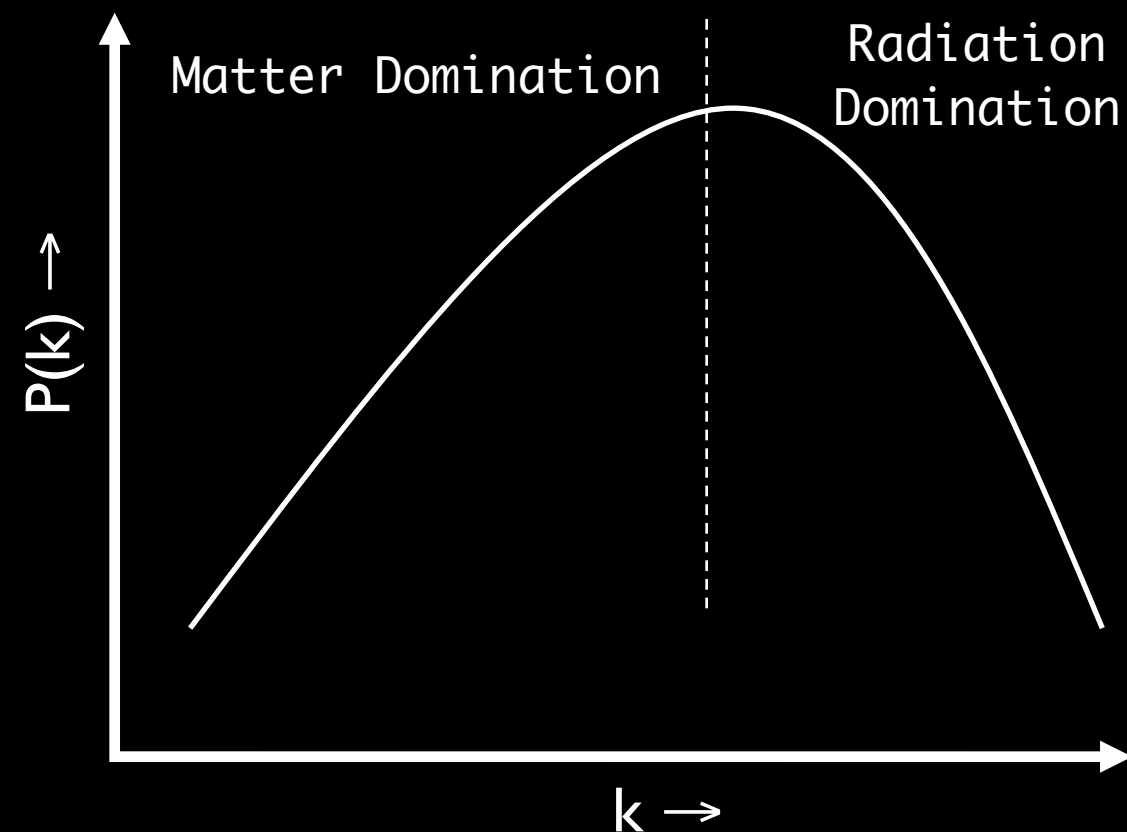
graphic from NASA/WMAP

Neutrinos?

Σm_ν — Σm_ν
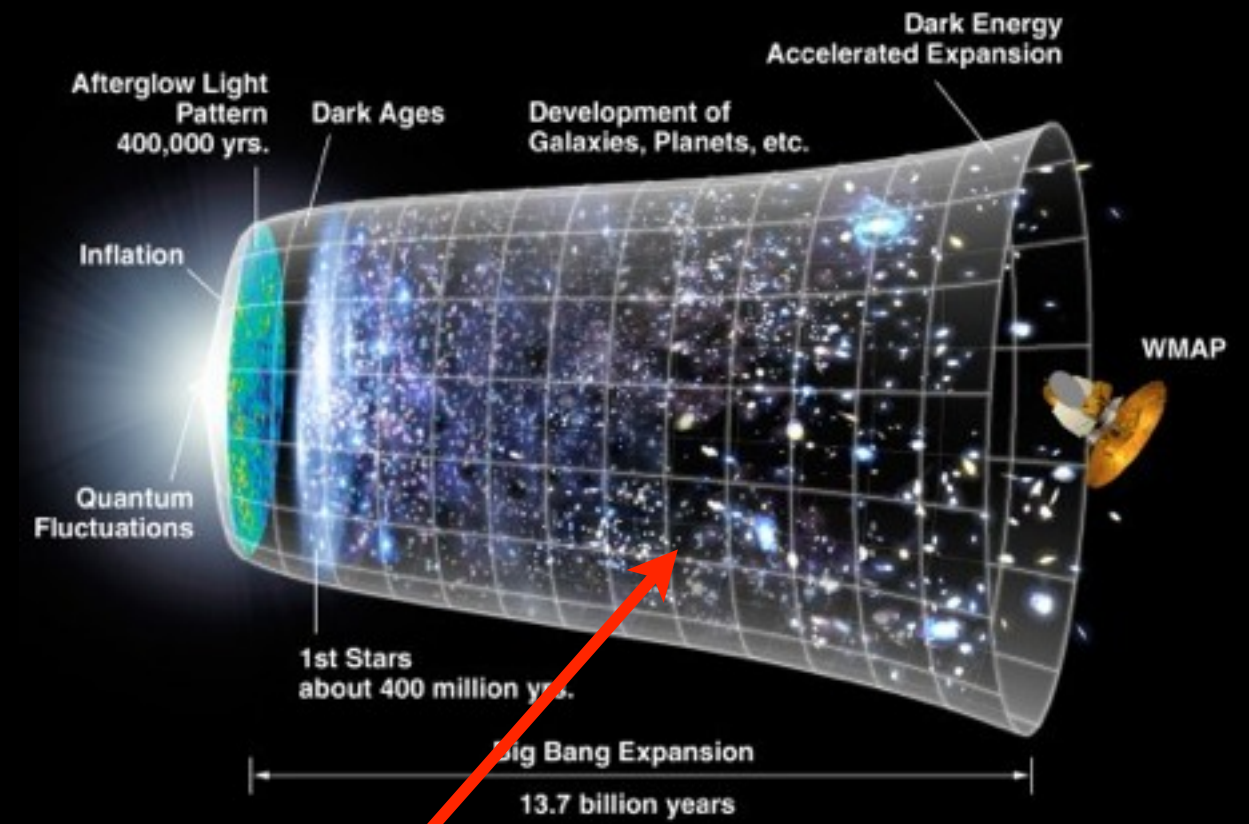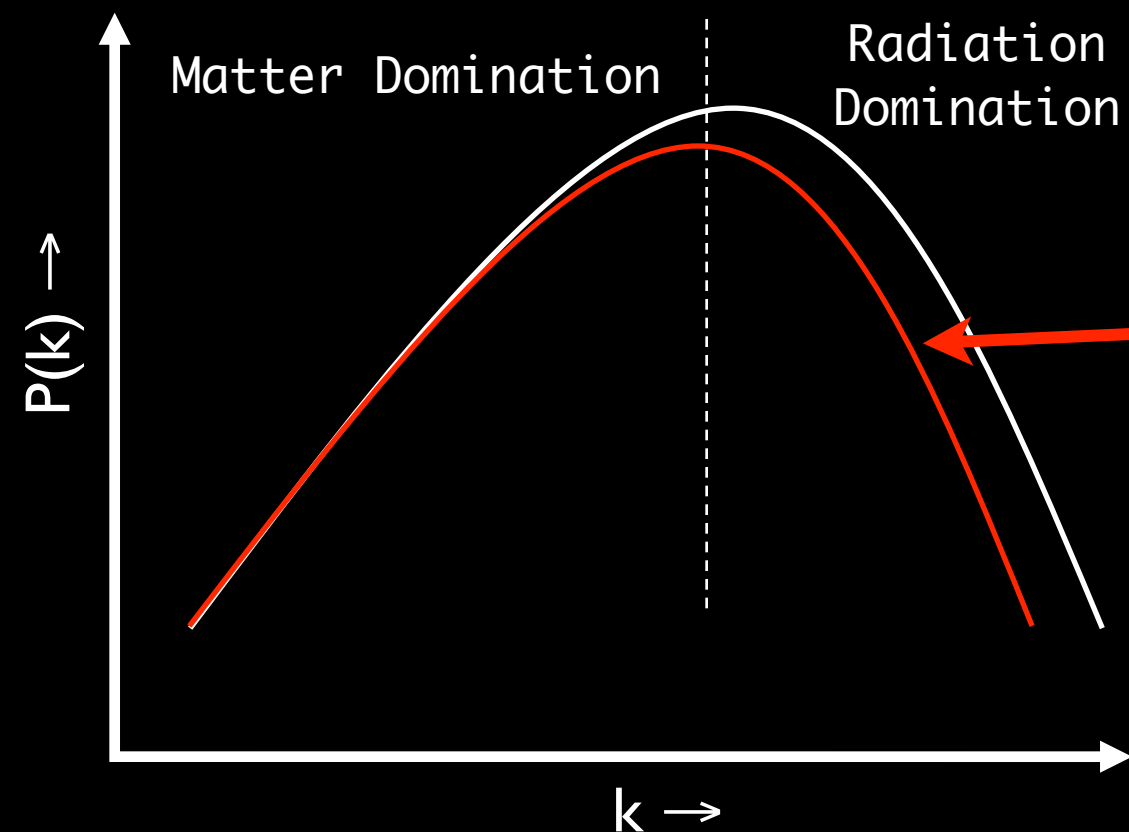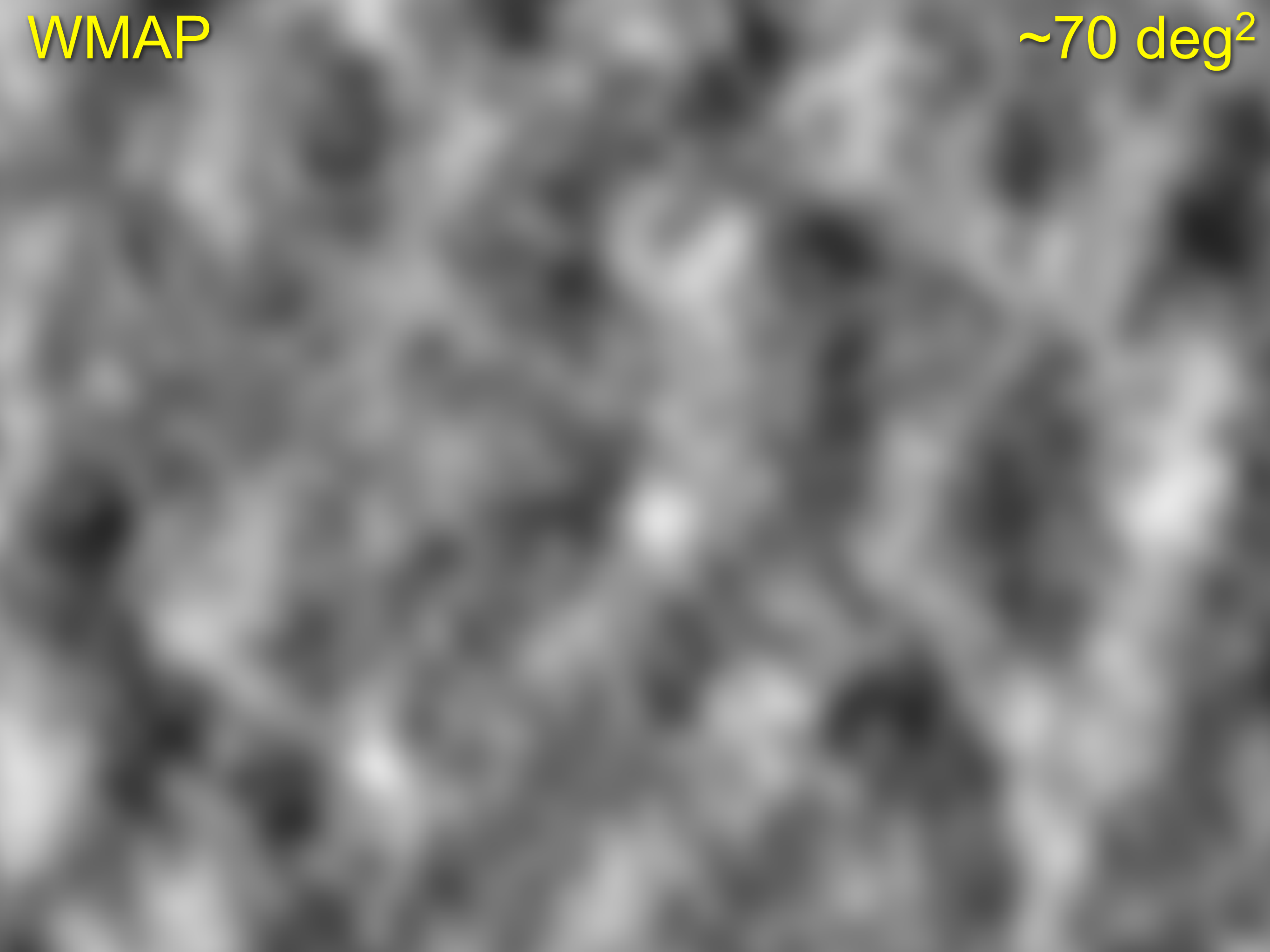
Sum of the neutrino masses
impacts growth of large scale structure,
i.e., the matter power spectrum
Probed by CMB lensing

Matter Domination

Radiation
Domination

$P(k) \rightarrow$

$k \rightarrow$

Afterglow Light
Pattern
400,000 yrs.

Dark Ages

Development of
Galaxies, Planets, etc.

Dark Energy
Accelerated Expansion

Inflation

Quantum
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WMAP

1st Stars
about 400 million yrs.

Big Bang Expansion

13.7 billion years

Neutrinos?

Matter Domination

Radiation Domination

P(k) →

k →

$\Sigma m_\nu > 0$

Sum of the neutrino masses impacts growth of large scale structure, i.e., the matter power spectrum
Probed by CMB lensing

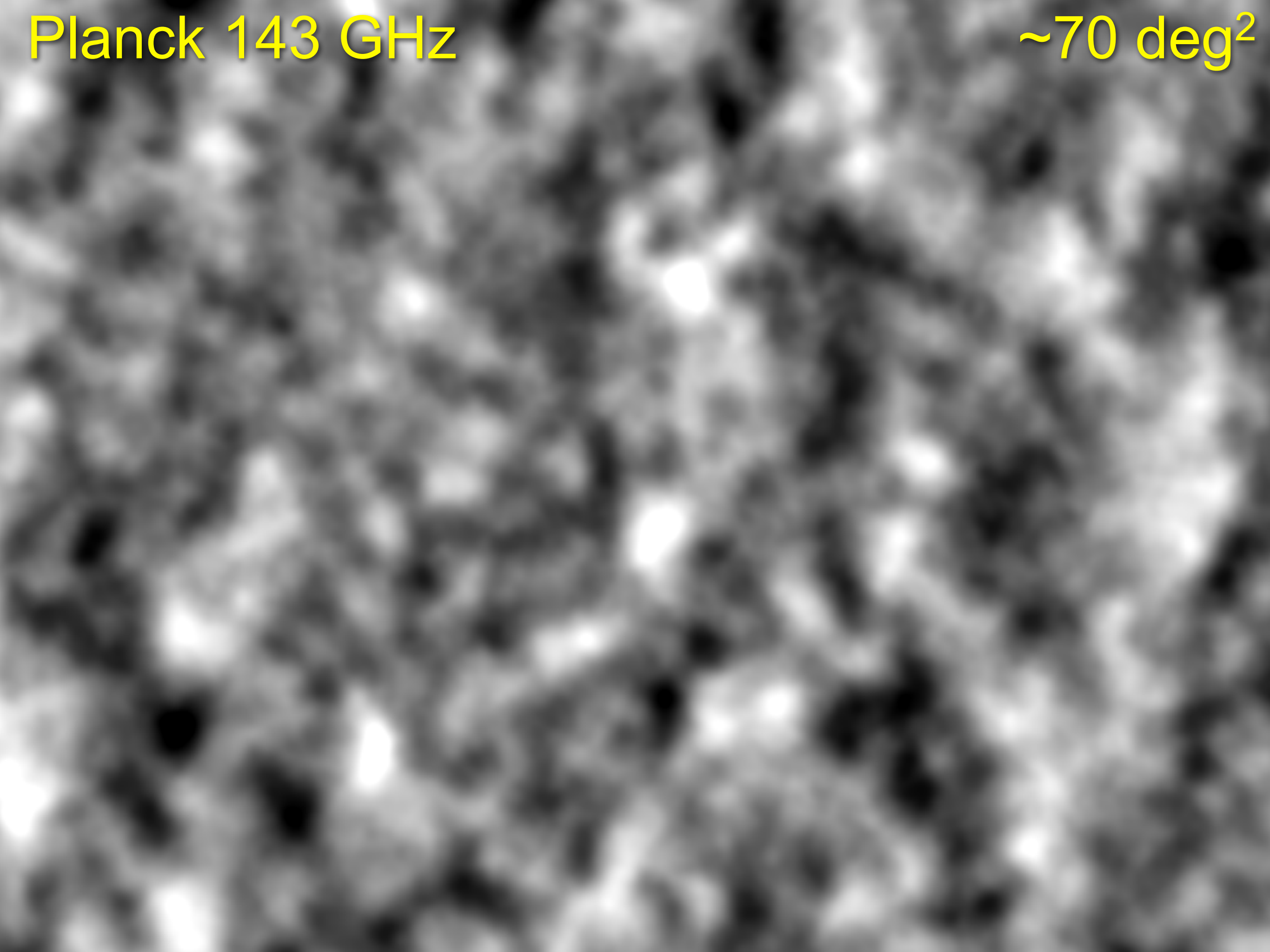Dark Energy
Accelerated Expansion

Afterglow Light
Pattern
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Dark Ages

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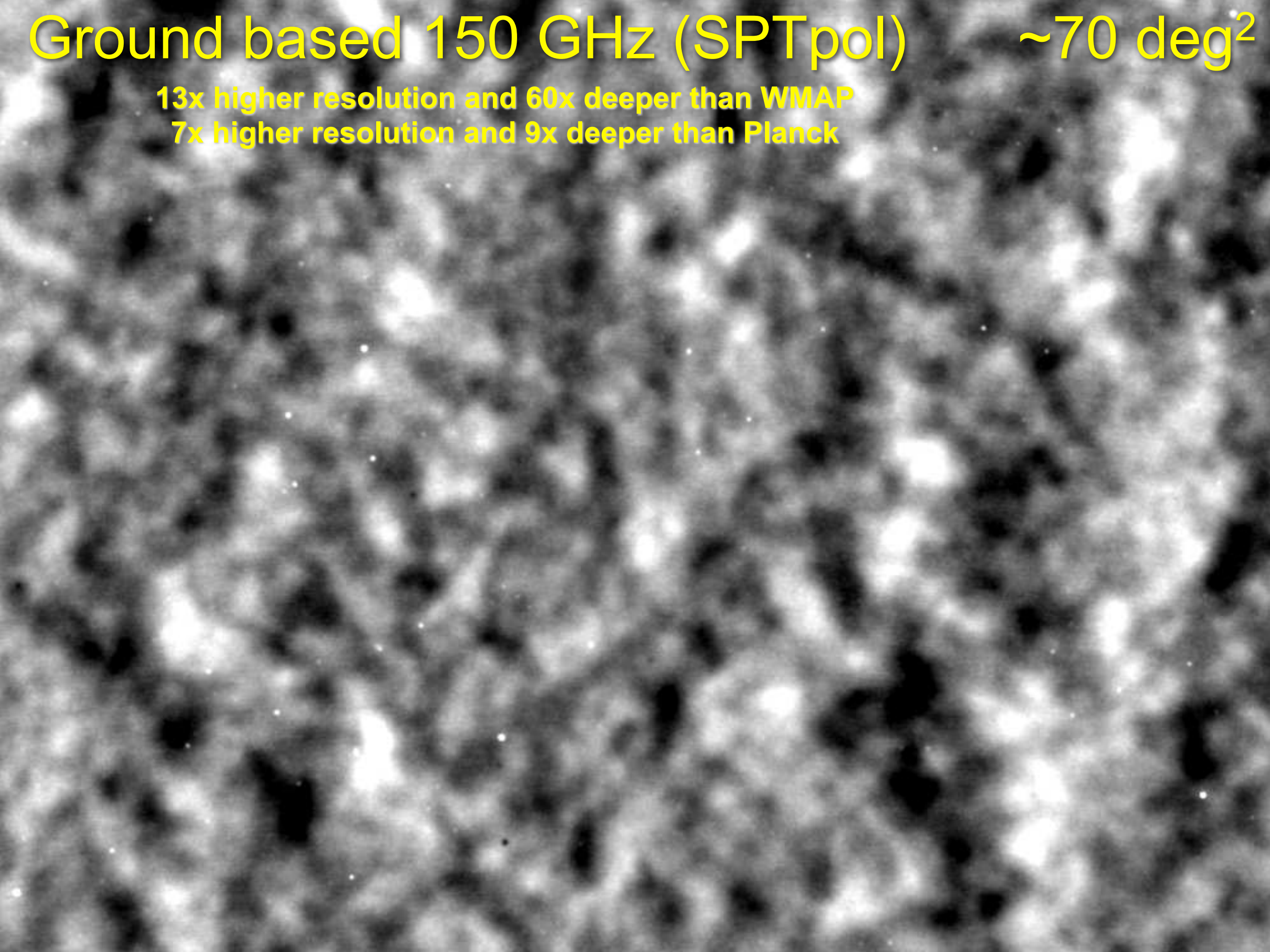
graphic from NASA/WMAP

Ground based 150 GHz (SPTpol)   ~70 deg^2
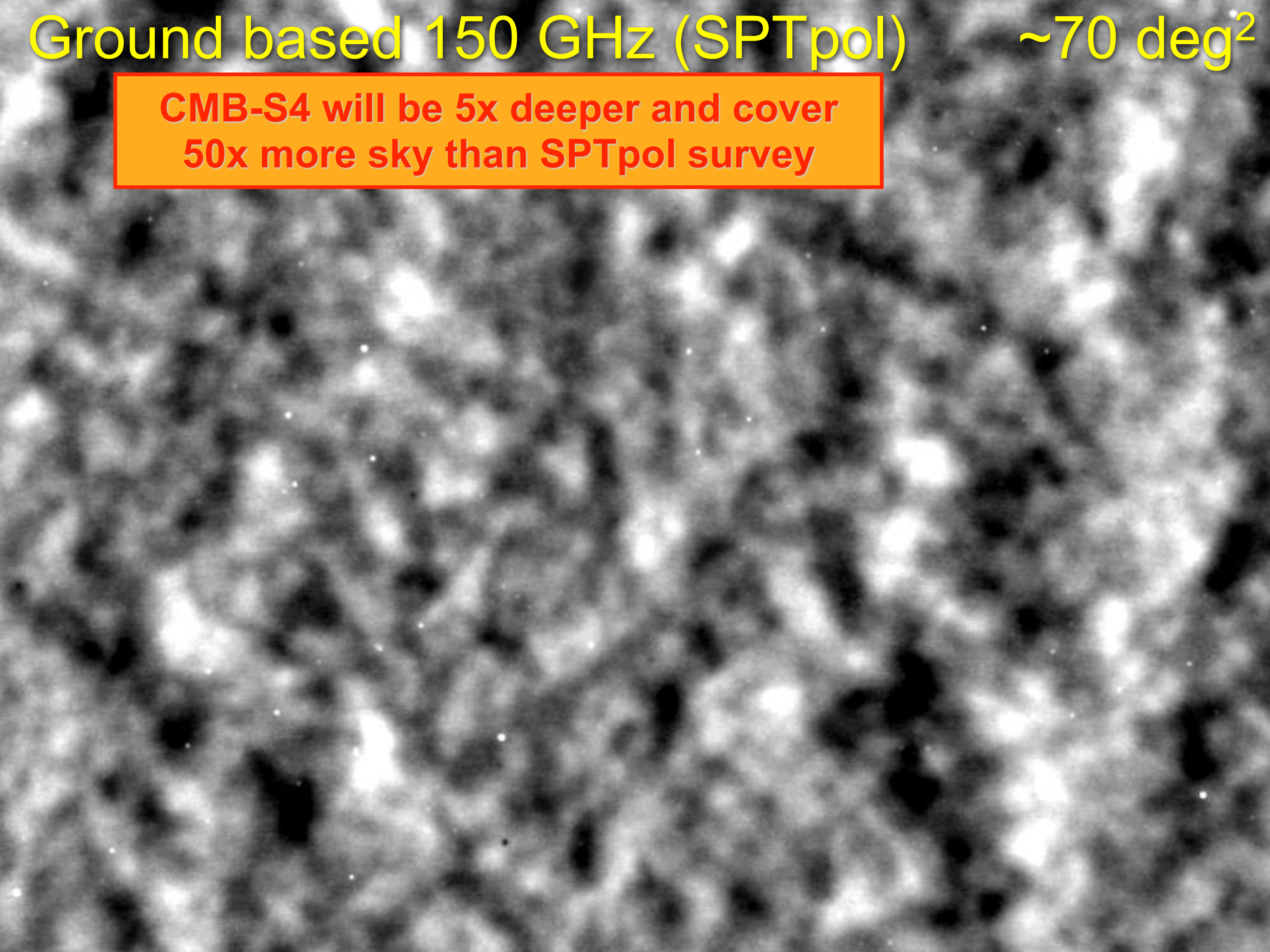
13x higher resolution and 60x deeper than WMAP
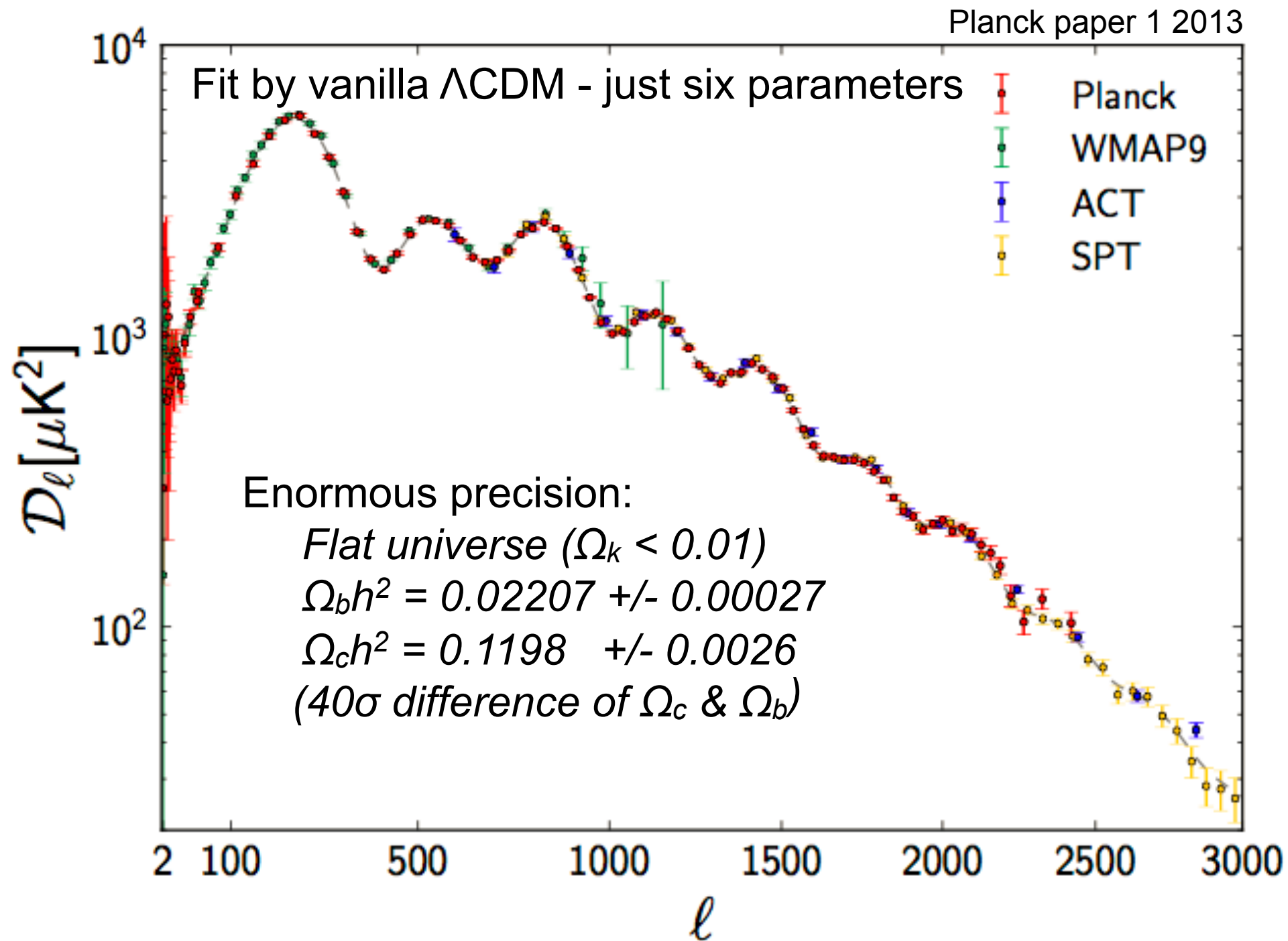7x higher resolution and 9x deeper than Planck

Ground based 150 GHz (SPTpol)   ~70 deg^2

CMB-S4 will be 5x deeper and cover
50x more sky than SPTpol survey

Primary CMB anisotropy - 9 harmonics

Planck paper 1 2013

Fit by vanilla ΛCDM - just six parameters

Enormous precision:
  Flat universe ($\Omega_k < 0.01$)
  $\Omega_b h^2 = 0.02207$ +/- 0.00027
  $\Omega_c h^2 = 0.1198$ +/- 0.0026
  (40σ difference of Ω_c & Ω_b)

Legend: Planck, WMAP9, ACT, SPT

Axes: $\mathcal{D}_\ell [\mu K^2]$ vs ℓ

Inflation checks: Geometrically flat universe; Superhorizon features; Acoustic peaks/adiabatic fluctuations; Departure from scale invariance.
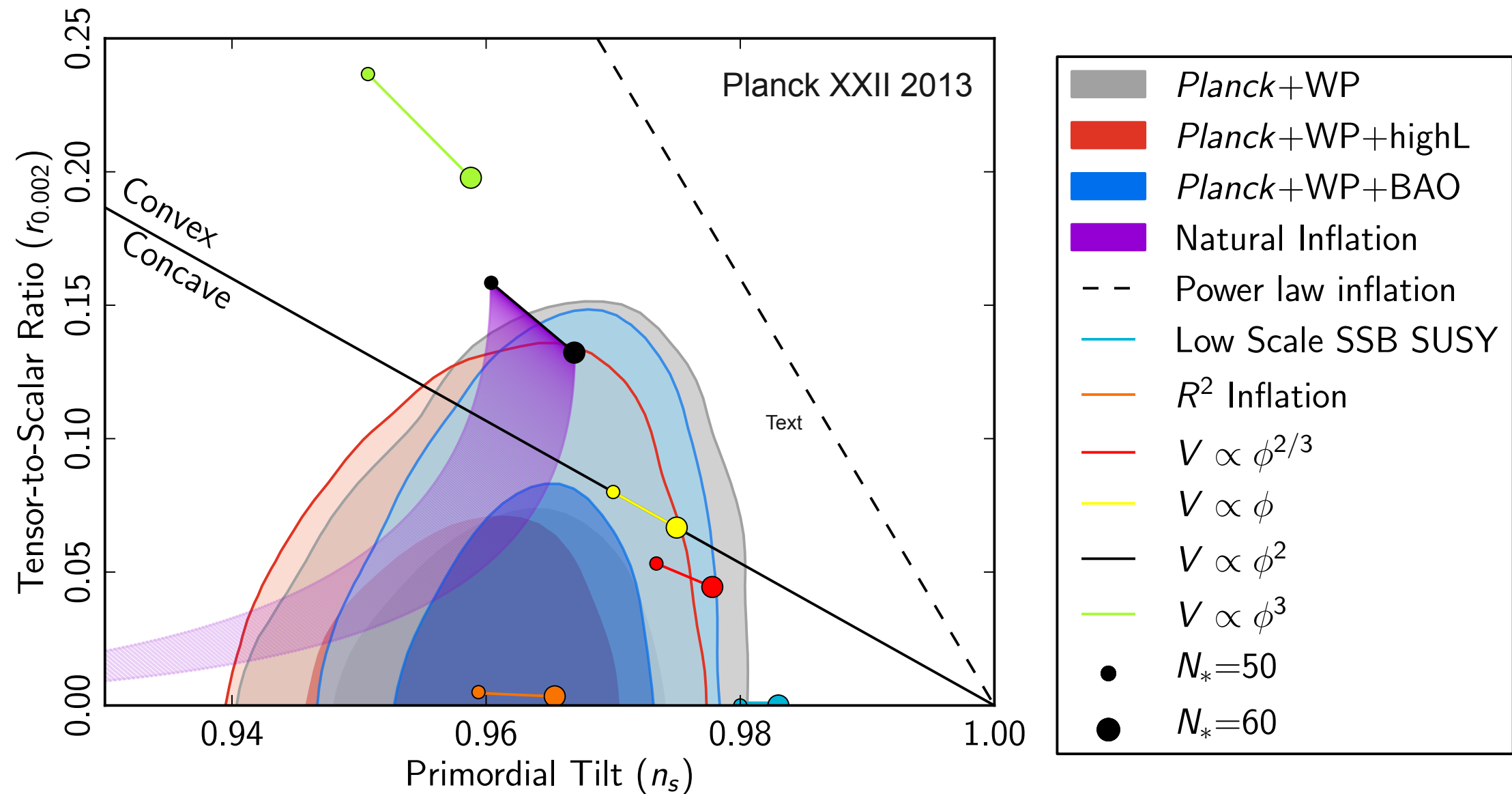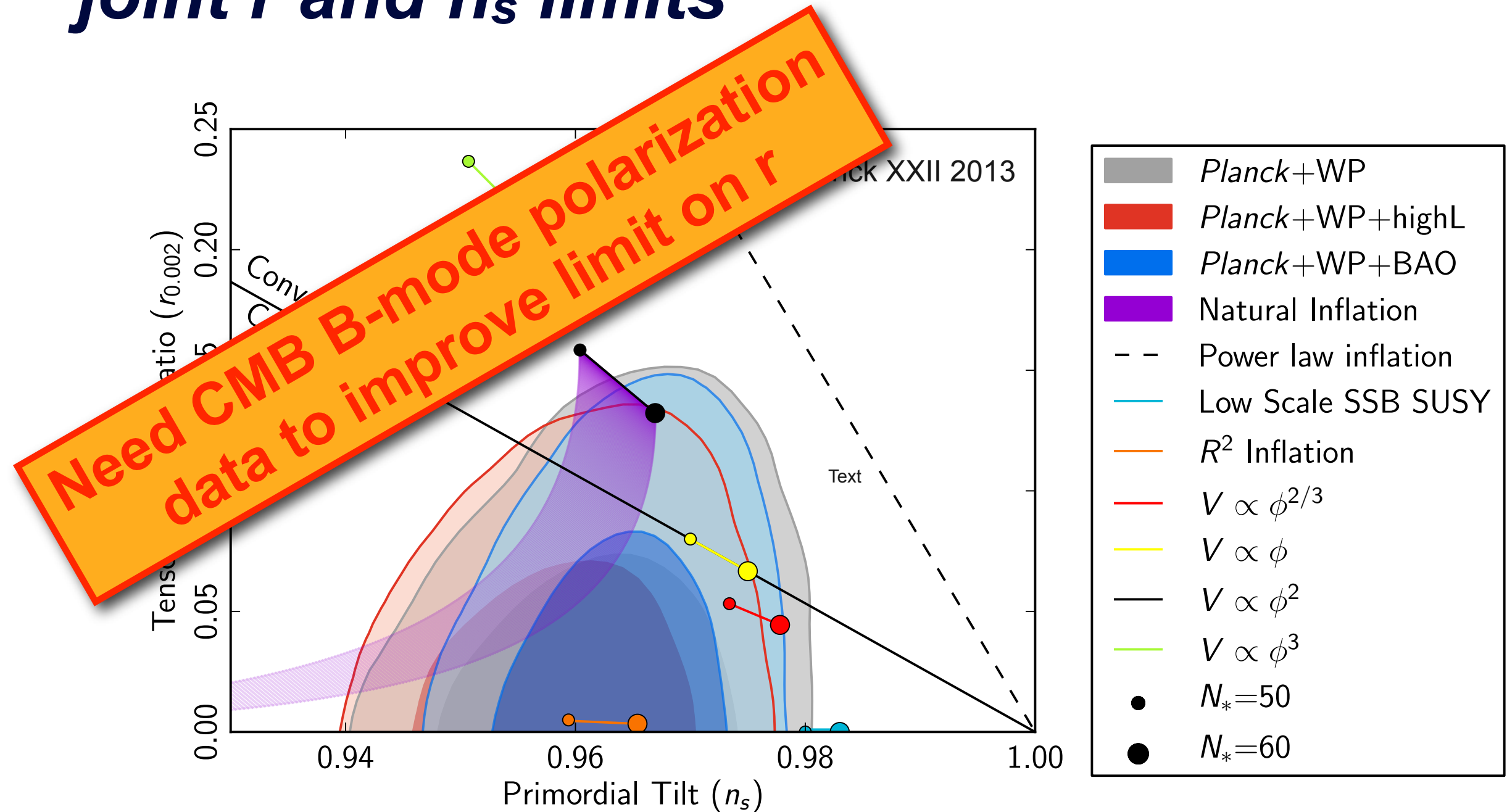
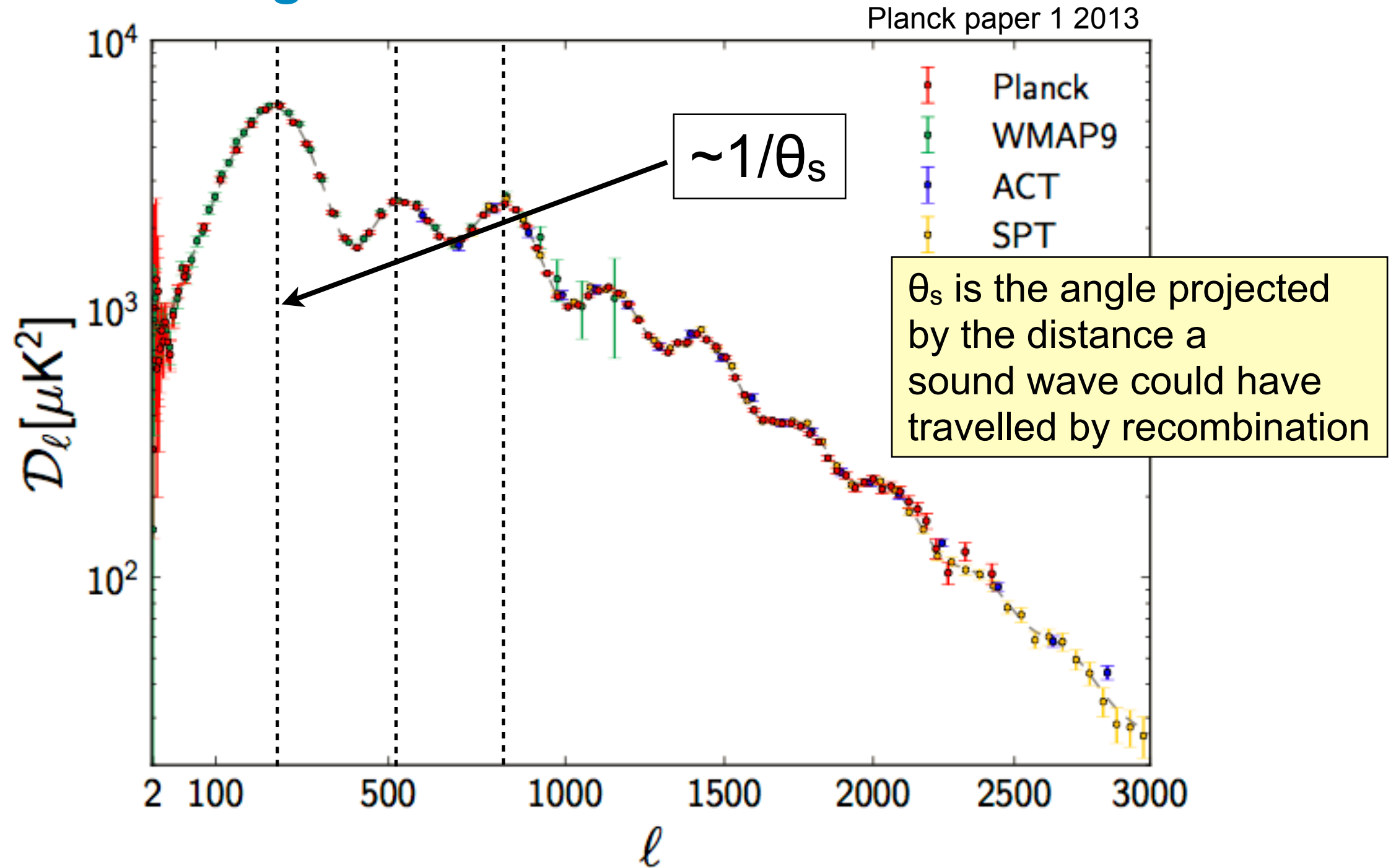# Constraining inflationary models
joint r and n_s limits

Spectral Index of primordial fluctuations, n_s,
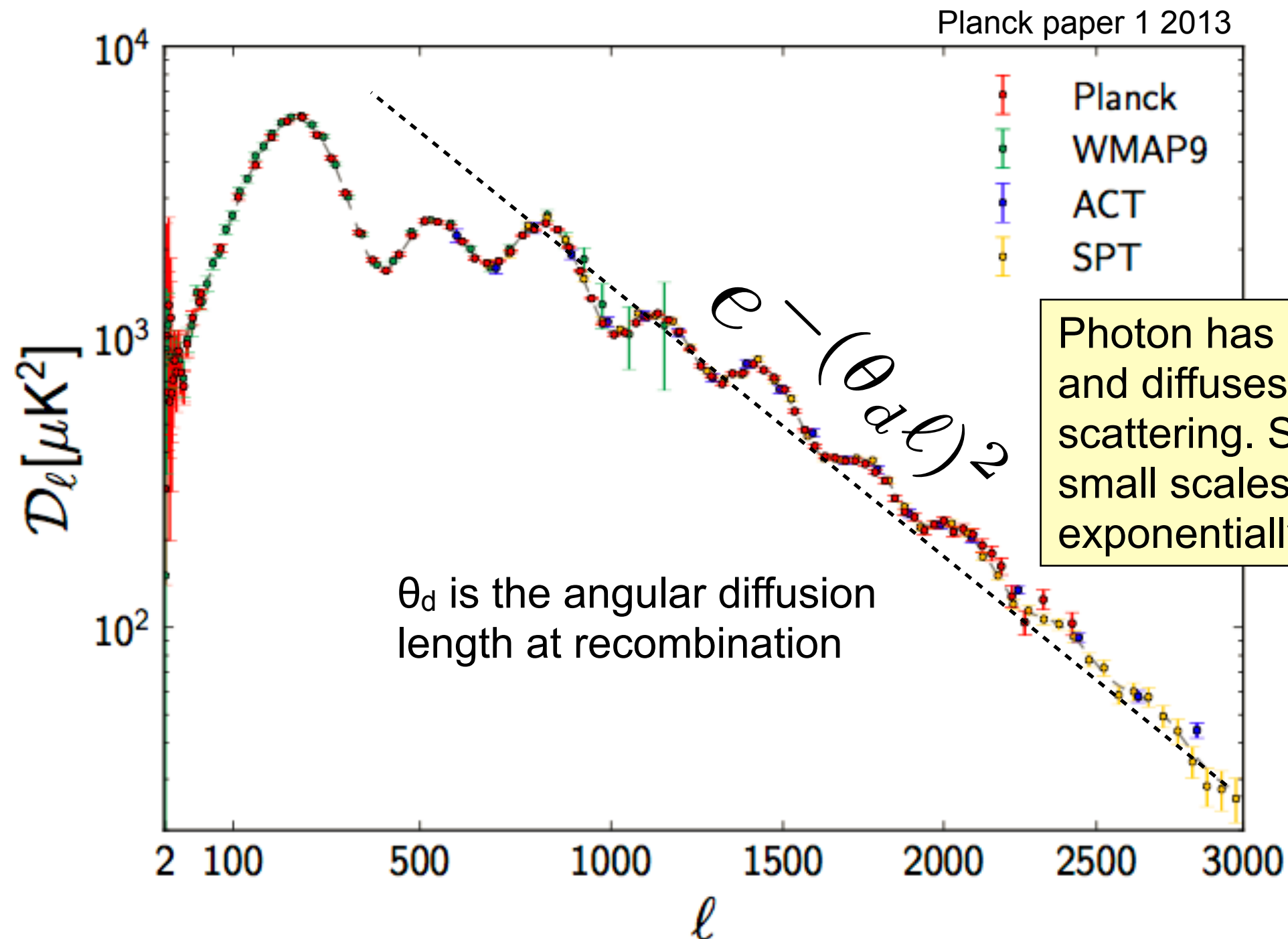where $\Delta_R^2(k) = \Delta_R^2(k_0)\left(\dfrac{k}{k_0}\right)^{n_s-1}$

Inflation evidence
$n_s \neq 1$ at over 5σ

Constraining inflationary models
joint r and n_s limits

Need CMB B-mode polarization data to improve limit on r

Planck XXII 2013

Legend:
- Planck+WP
- Planck+WP+highL
- Planck+WP+BAO
- Natural Inflation
- Power law inflation
- Low Scale SSB SUSY
- R^2 Inflation
- $V \propto \phi^{2/3}$
- $V \propto \phi$
- $V \propto \phi^2$
- $V \propto \phi^3$
- $N_* = 50$
- $N_* = 60$

Axis labels: Tensor-to-Scalar Ratio ($r_{0.002}$); Primordial Tilt (n_s)

Spectral Index of primordial fluctuations, n_s,
where $\Delta_R^2(k) = \Delta_R^2(k_0)\left(\dfrac{k}{k_0}\right)^{n_s-1}$

Inflation evidence
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Primary CMB anisotropy - 9 harmonics
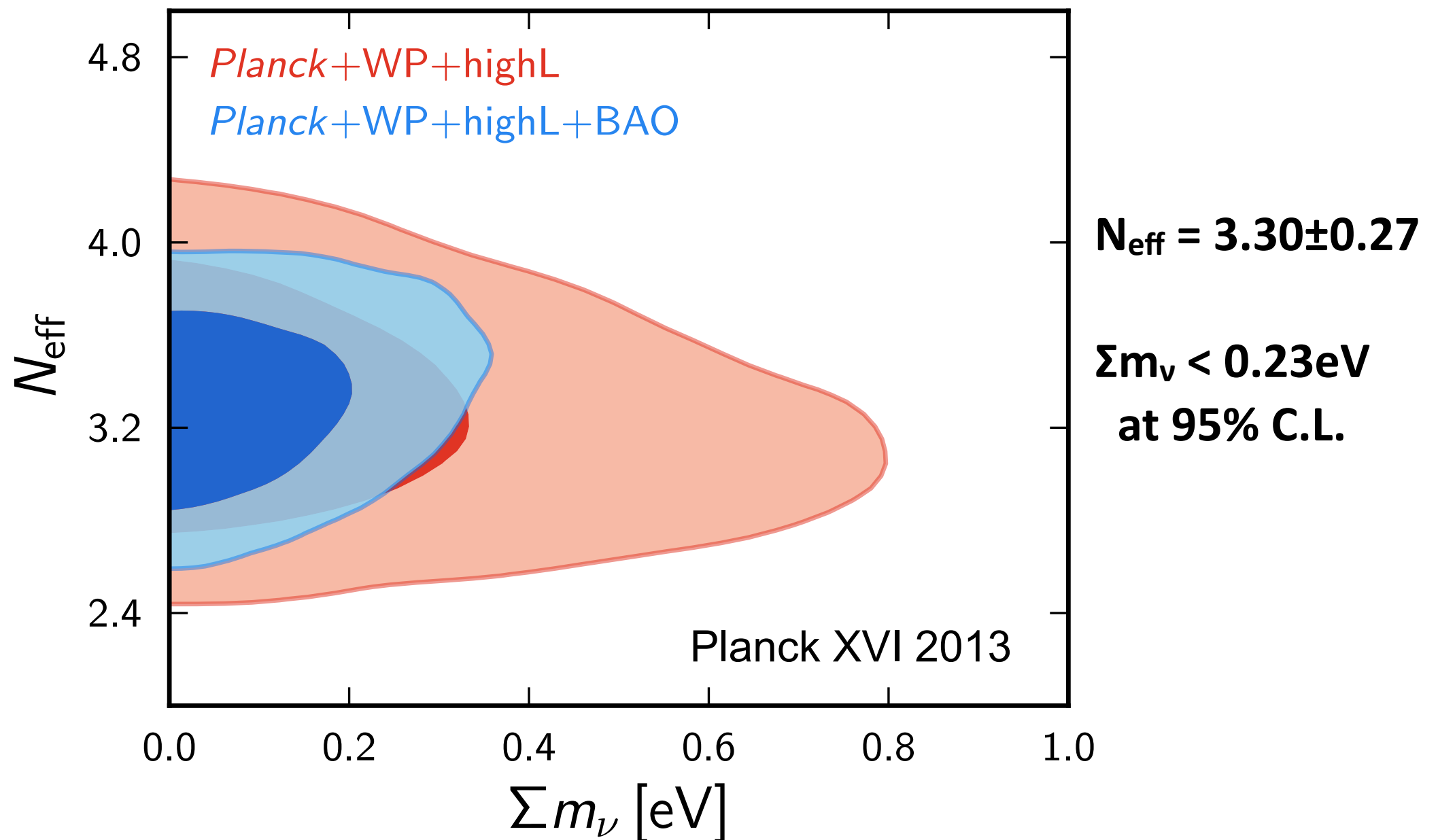## Improves precision of sound horizon, θ_s, & provides larger lever arm

Planck paper 1 2013

~$1/\theta_s$

θ_s is the angle projected by the distance a sound wave could have travelled by recombination

And most importantly provides determination of the damping scale, θ_d

Planck paper 1 2013

Legend:
- Planck
- WMAP9
- ACT
- SPT

$e^{-(\theta_d \ell)^2}$

Photon has a mean free path and diffuses prior to last scattering. So, oscillations on small scales are damped exponentially. (Silk damping)

θ_d is the angular diffusion length at recombination

Note $\dfrac{r_d}{r_s} = \dfrac{\theta_d}{\theta_s} \propto H^{0.5}$ so ratio is sensitive to energy density.

Constraining model extensions:
joint N_{eff} and Σm_ν constraints

Planck+WP+highL
Planck+WP+highL+BAO

$N_{eff} = 3.30 \pm 0.27$
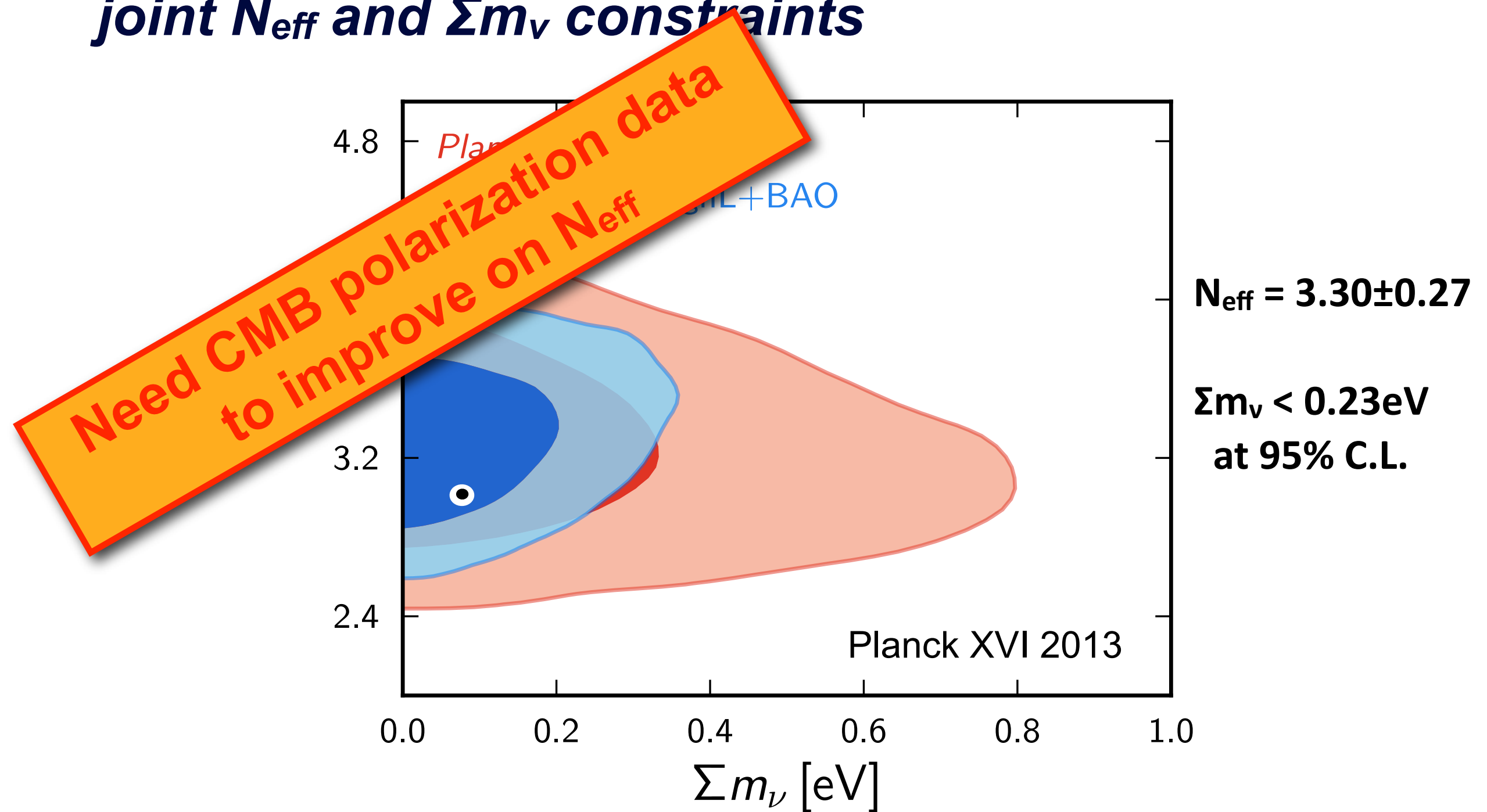
$\Sigma m_\nu < 0.23$eV
at 95% C.L.

Planck XVI 2013

N_{eff} is the effective number of relativistic species.
For standard 3 neutrinos N_{eff} =3.046.
It measures the extra energy relative to the photons.

Constraining model extensions:
joint N_{eff} and Σm_ν constraints

Need CMB polarization data
to improve on N_{eff}

$N_{eff} = 3.30 \pm 0.27$

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Planck XVI 2013

Σm_ν [eV]

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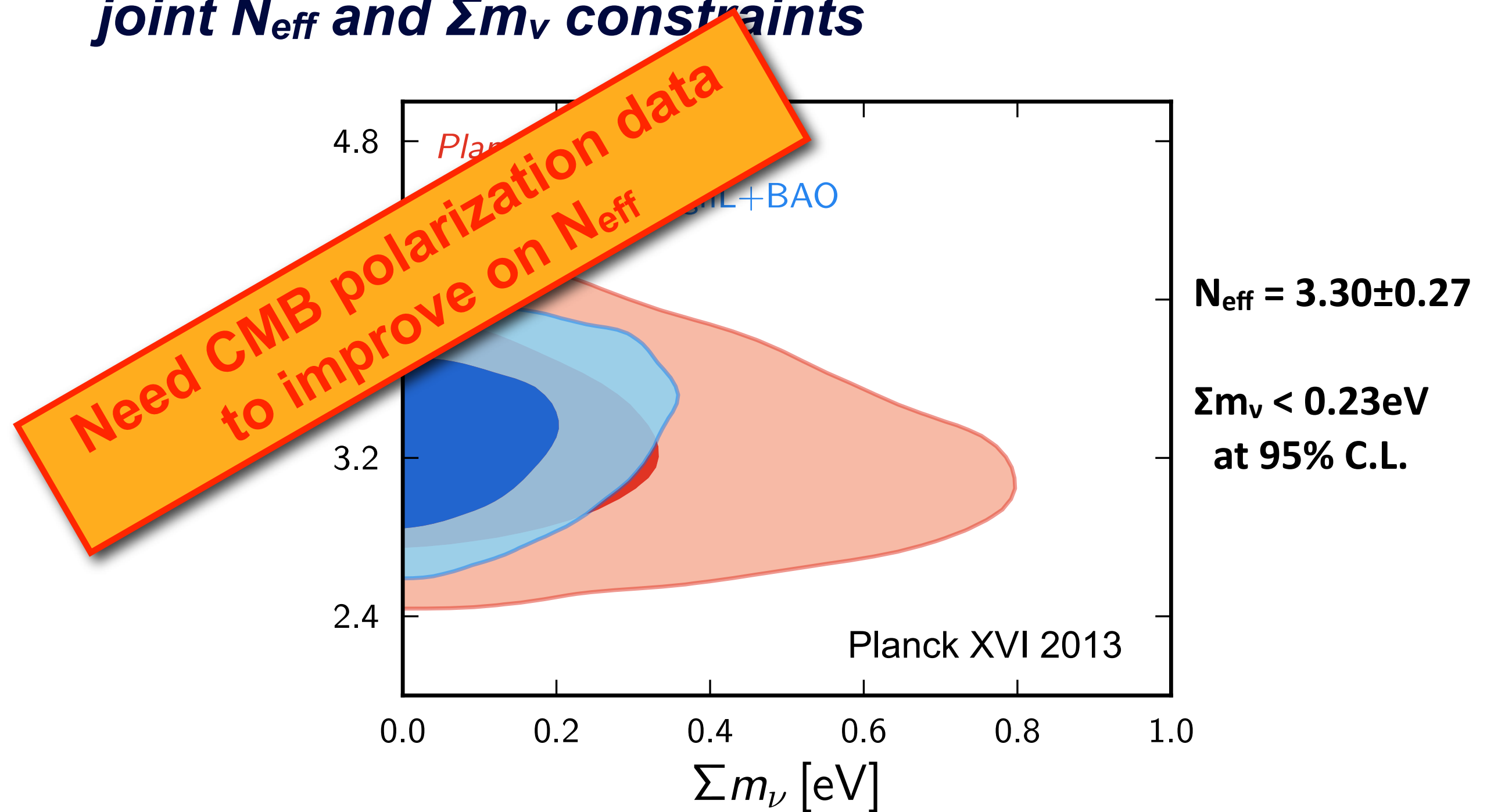
CMB lensing

Large-Scale Structure Lenses the CMB

- RMS deflection of ~2.5'
- Lensing efficiency peaks at $z \sim 2$
- Coherent on ~degree (~300 Mpc) scales

graphic from ESA Website

Lensing of the CMB

17°x17°
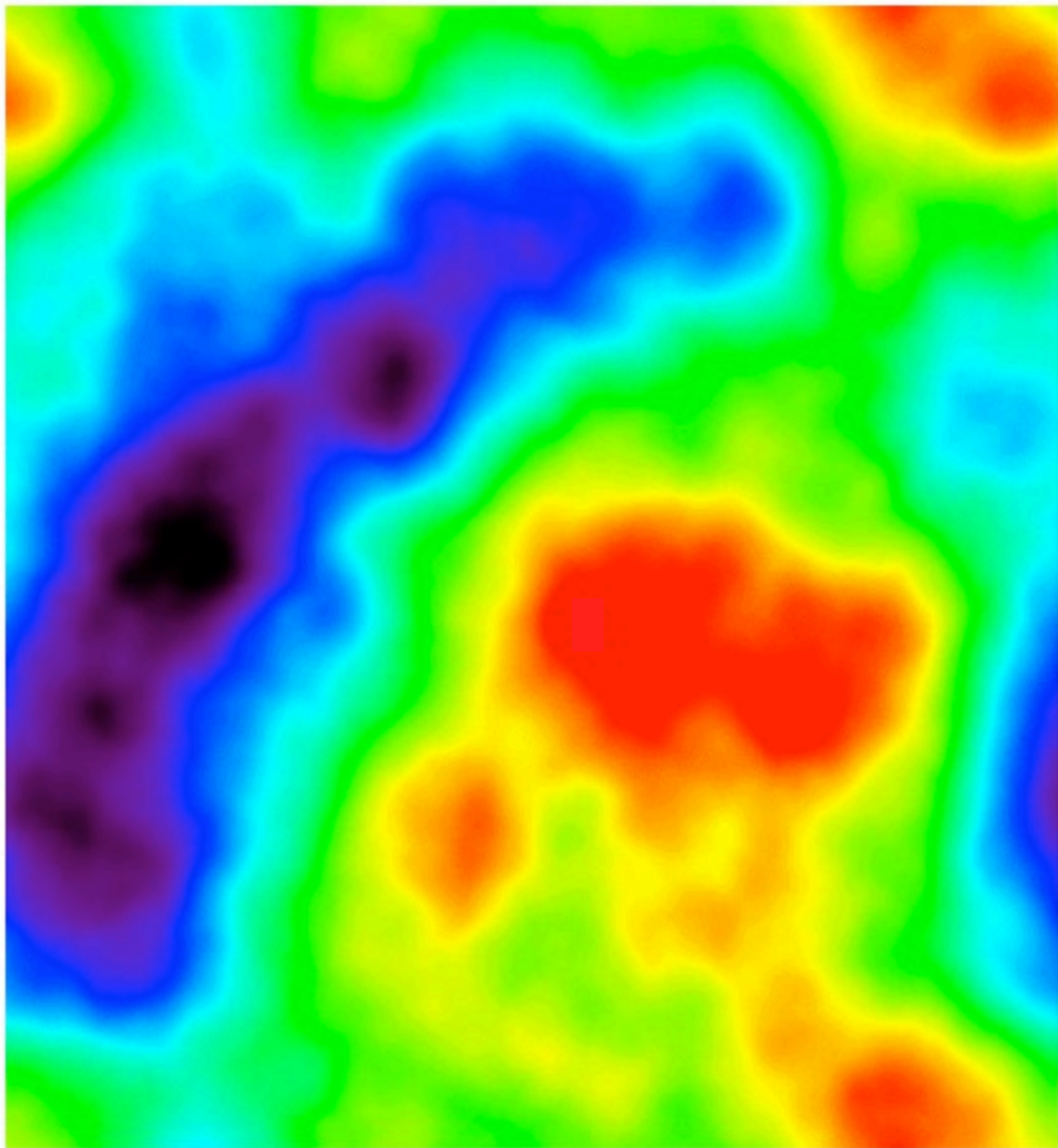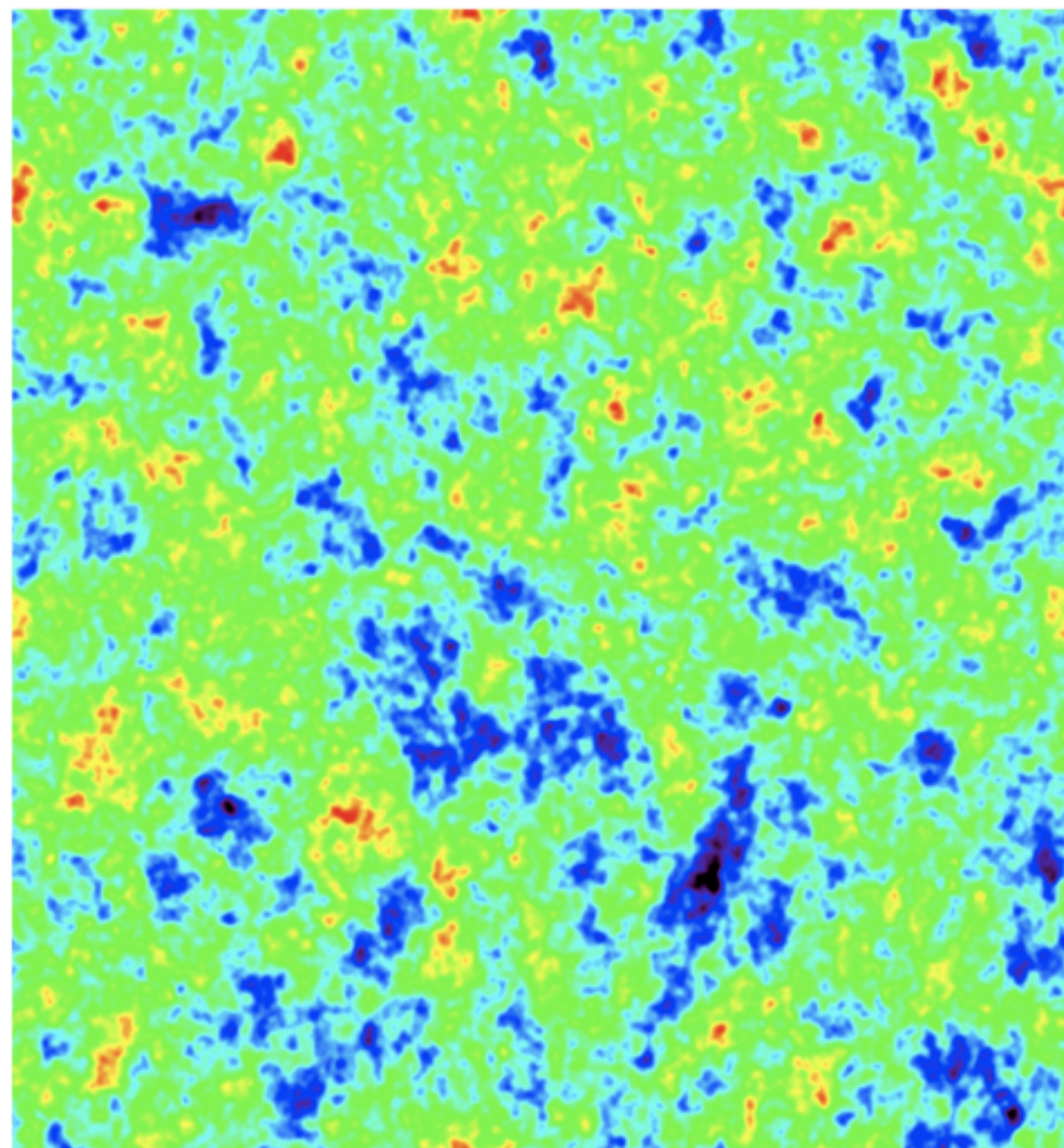
lensing potential
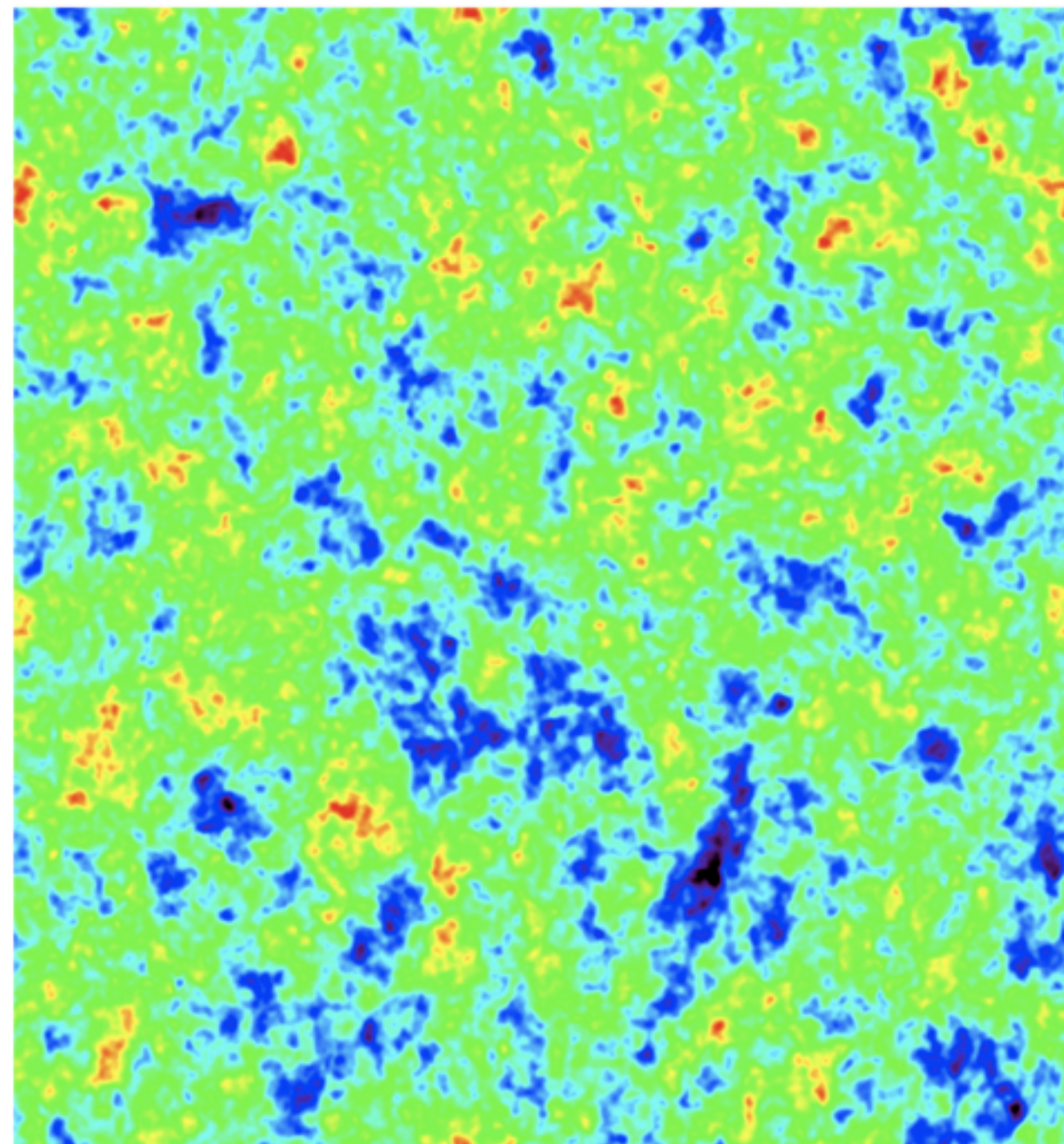
unlensed cmb

from Alex van Engelen

Lensing of the CMB

17°x17°
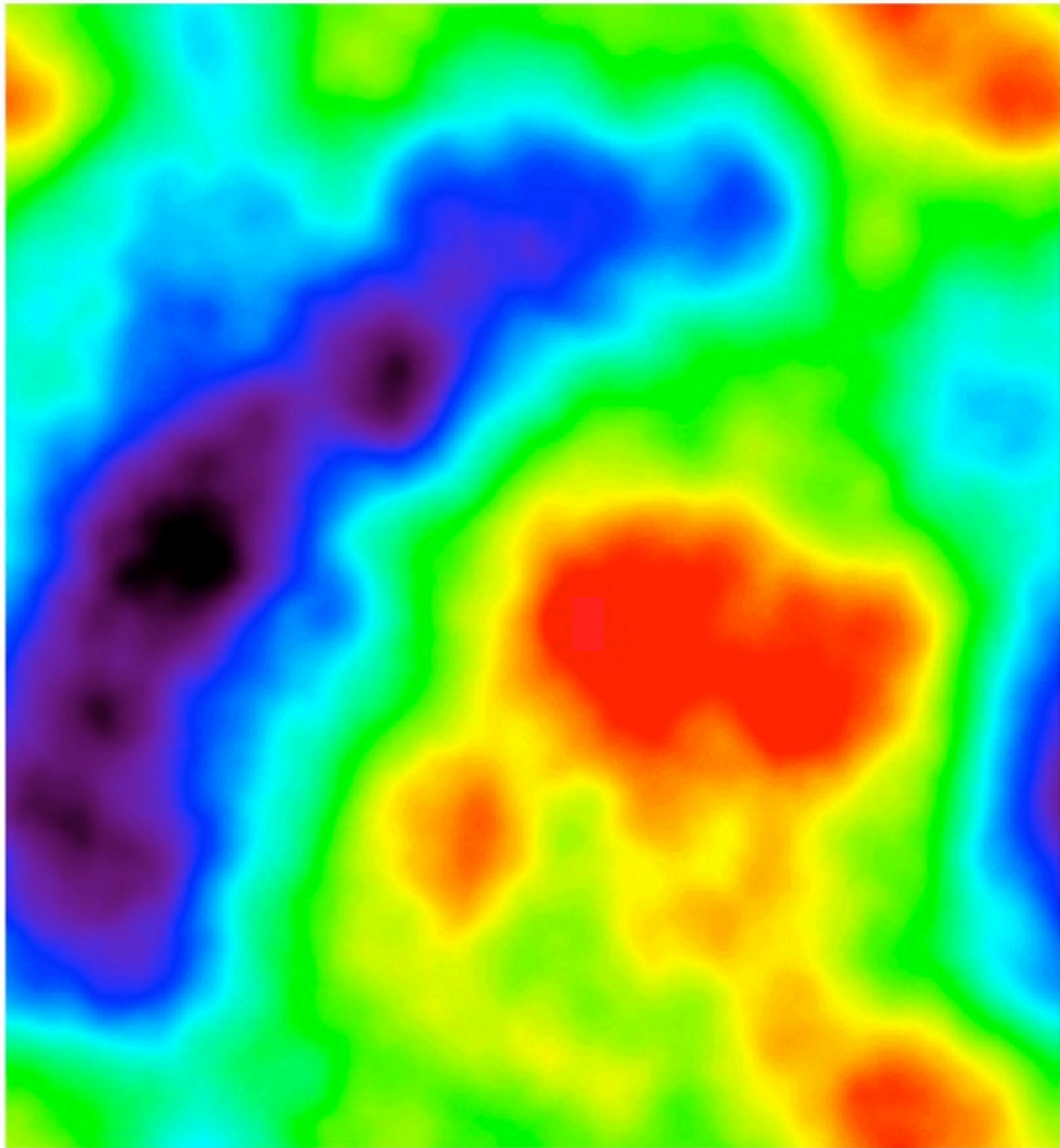
lensing potential

lensed cmb

from Alex van Engelen

We can take a CMB map
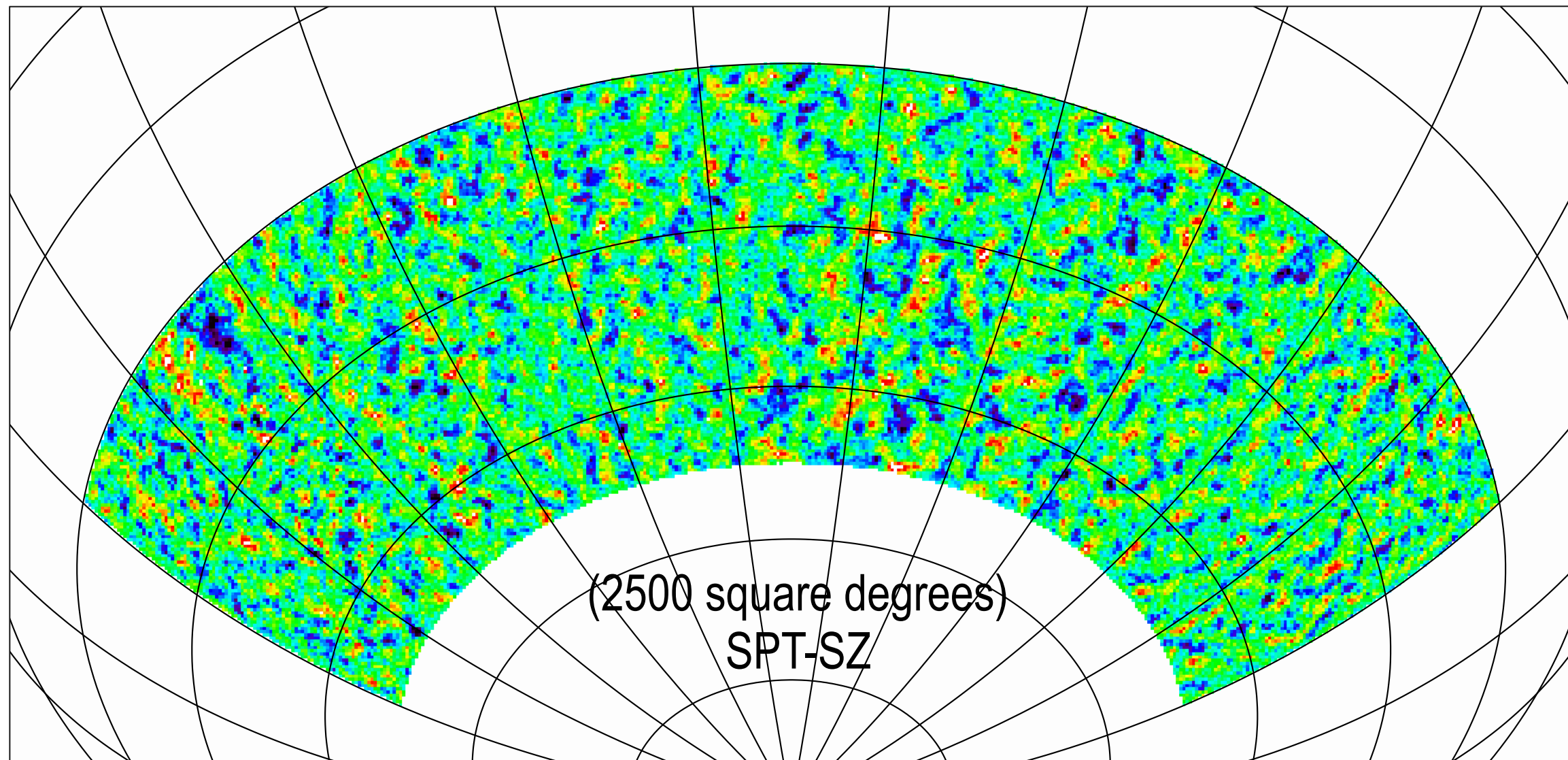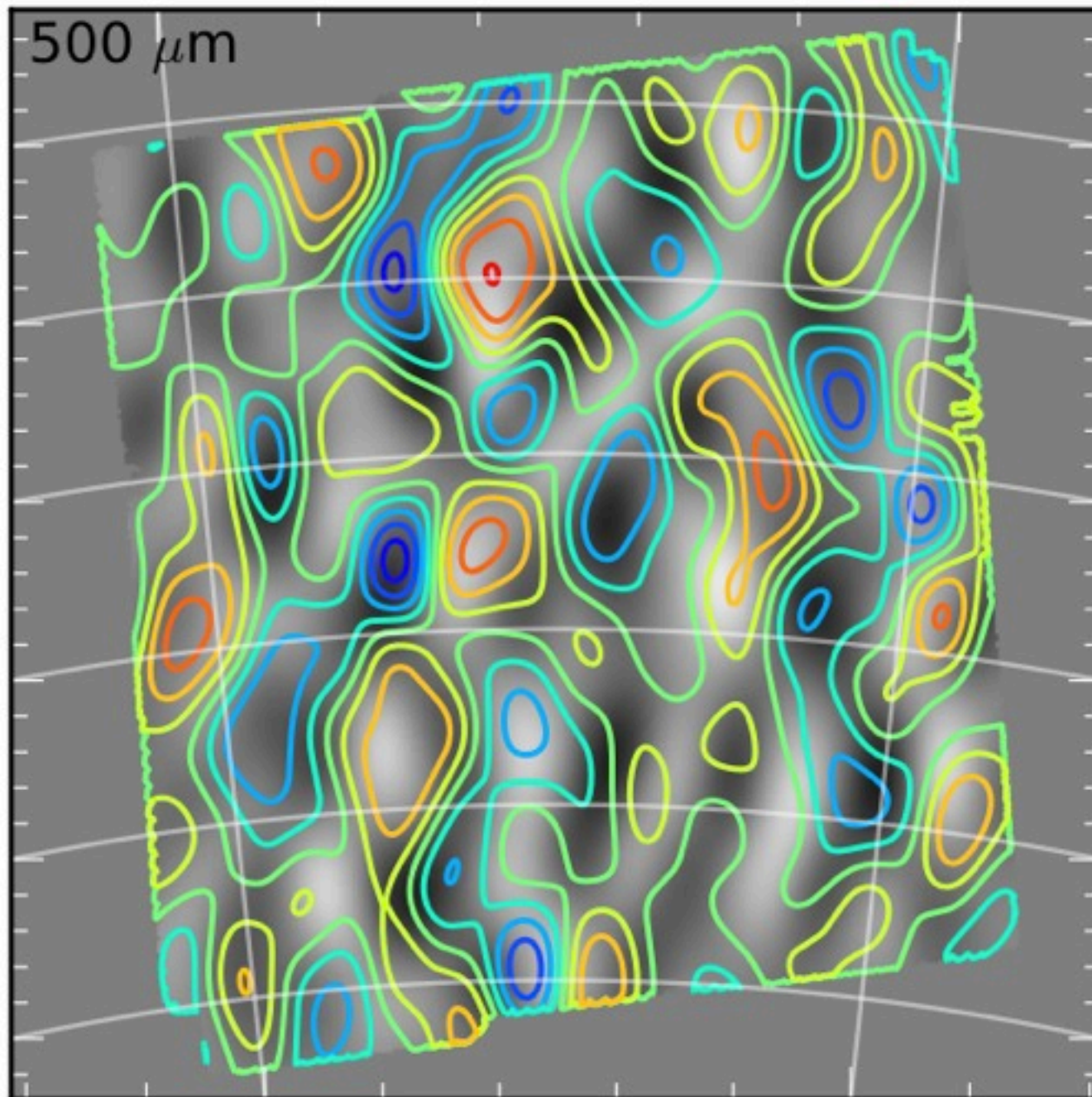

(2500 square degrees)
SPT-SZ

and construct the CMB Lensing map

**reconstruction of the mass projected
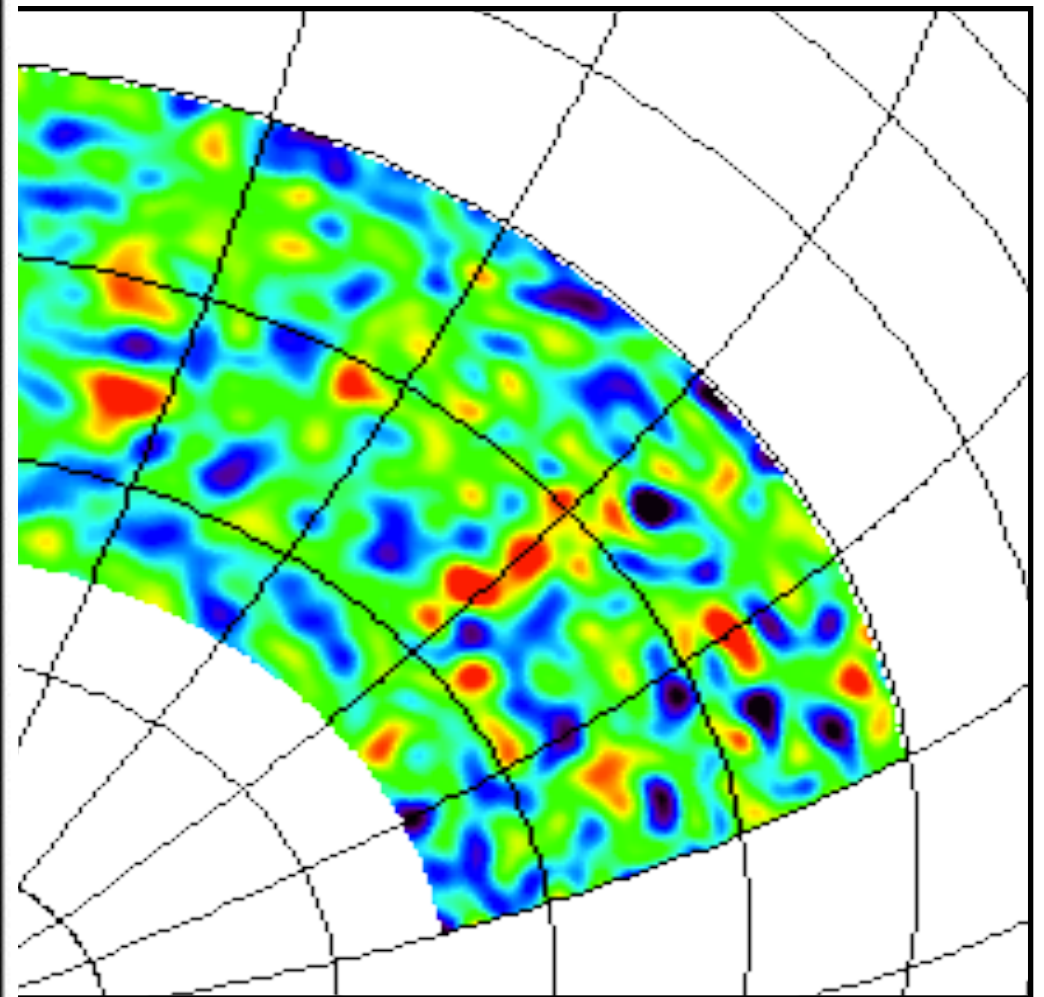along the line of sight to the CMB.**
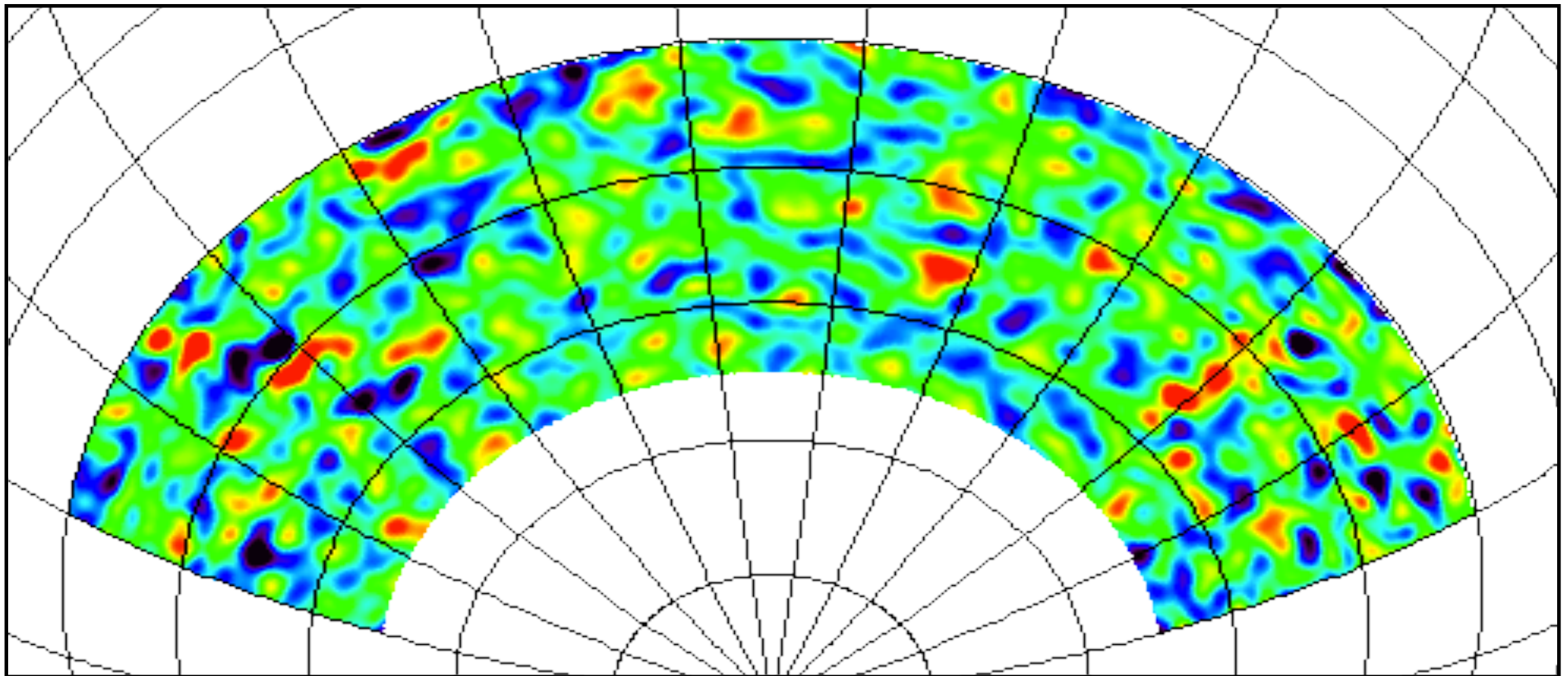
Lensing convergence map smoothed to 1 deg resolution

and construct the CMB Lensing map

reconstruction of the mass projected along the line of sight to the CMB.
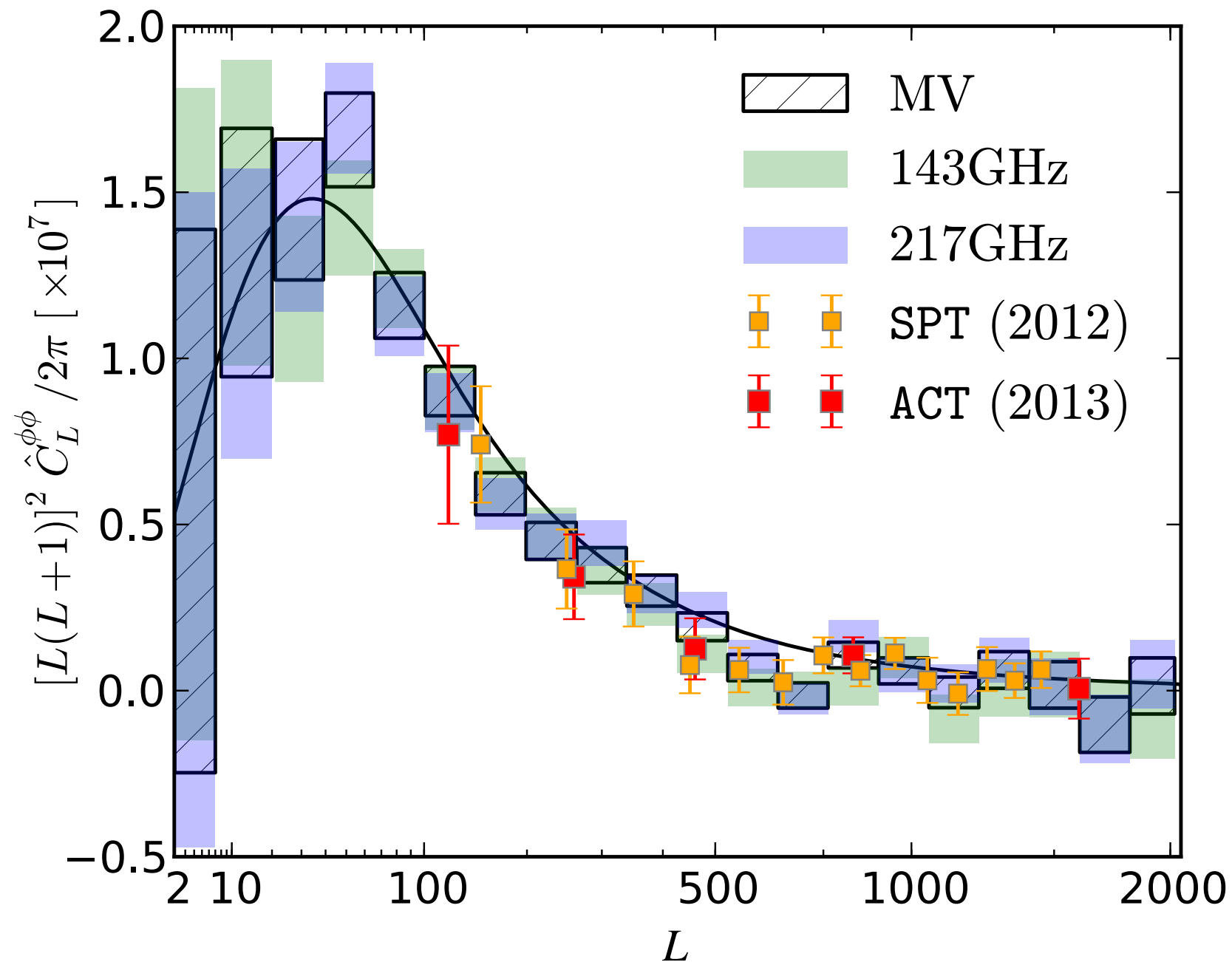

500 μm

Correlation of matter traced by CMB lensing (contours) and distribution of high z galaxies (grayscale; Herschel 500 um) [arXiv:1112.5435]

Lensing convergence map smoothed to 1 deg resolution
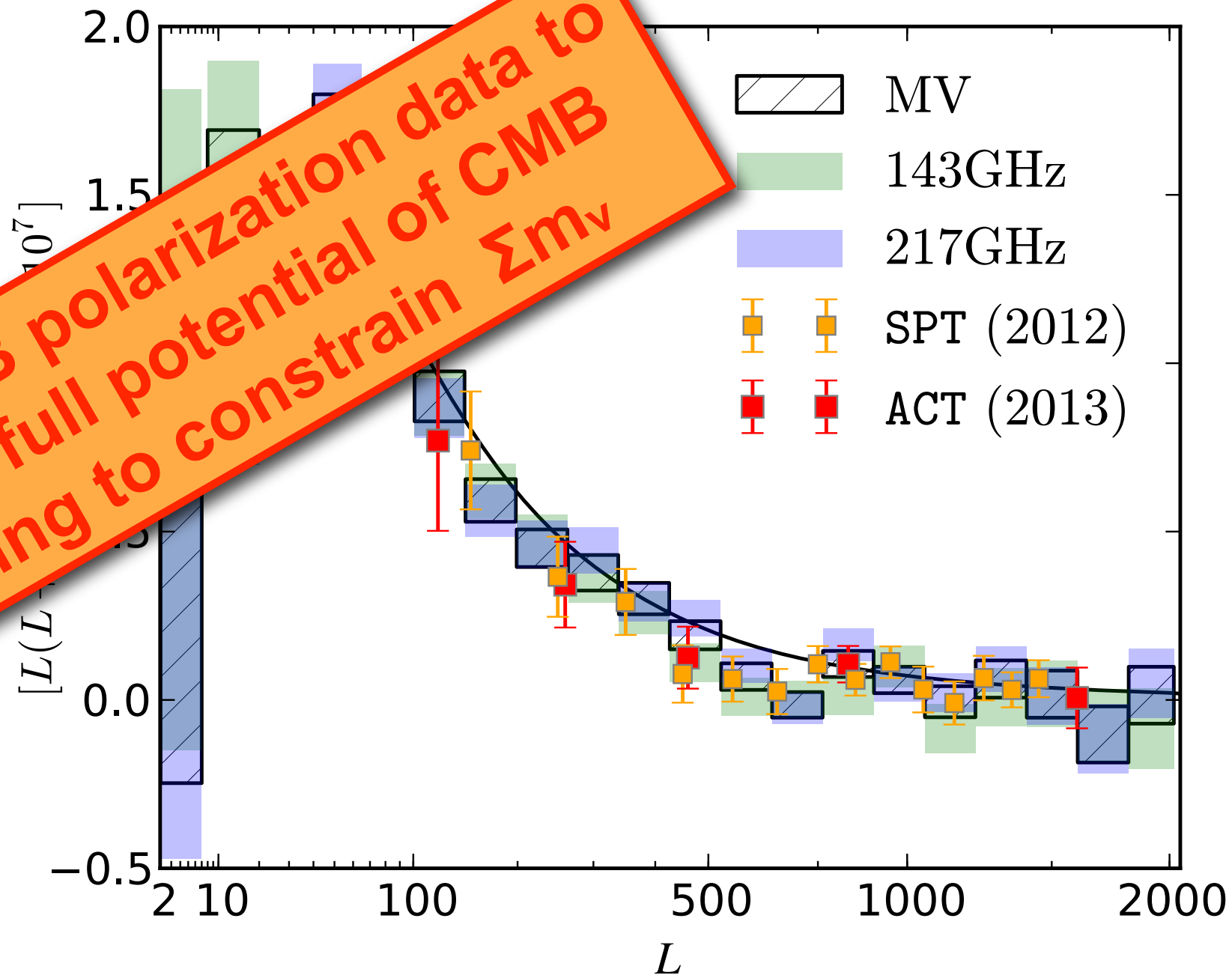
CMB lensing power spectrum

Planck XVII 2013

Sensitive to the neutrino masses

$\sum m_\nu$ = 0.1 eV → 5% amplitude of spectrum

Polarization gives additional lensing sensitivity and is a cleaner probe.

CMB lensing power spectrum

Planck XVII 2013

Need CMB polarization data to exploit full potential of CMB lensing to constrain Σm_ν

Legend:
- MV
- 143GHz
- 217GHz
- SPT (2012)
- ACT (2013)

Sensitive to the neutrino masses
$\sum m_\nu$ = 0.1 eV → 5% amplitude of spectrum

Polarization gives additional lensing sensitivity and is a cleaner probe.

CMB lensing and optical surveys

CMB lensing complements large optical surveys such as DES, eBOSS, LSST, DESI, Euclid, WFIRST, etc.

The combination leads to better shear-bias calibration and more robust constraints on Dark Energy and the properties of neutrinos.

→Critical for CMB-S4 sky coverage to overlap optical surveys.
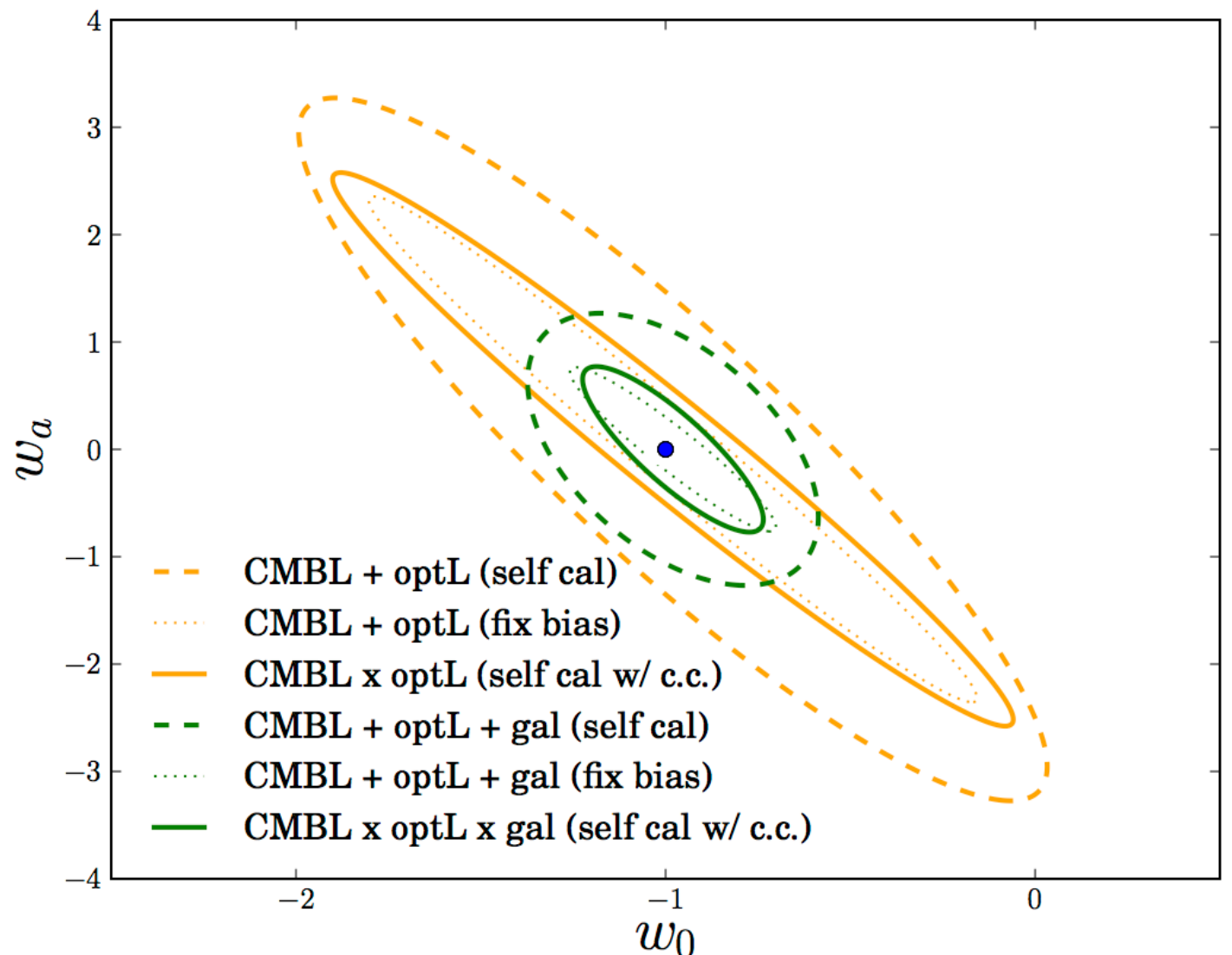
From "Can CMB Lensing Help Cosmic Shear Surveys?" Das, Errard, and Spergel, 2013

CMB polarization:
the next frontier for lensing & inflation
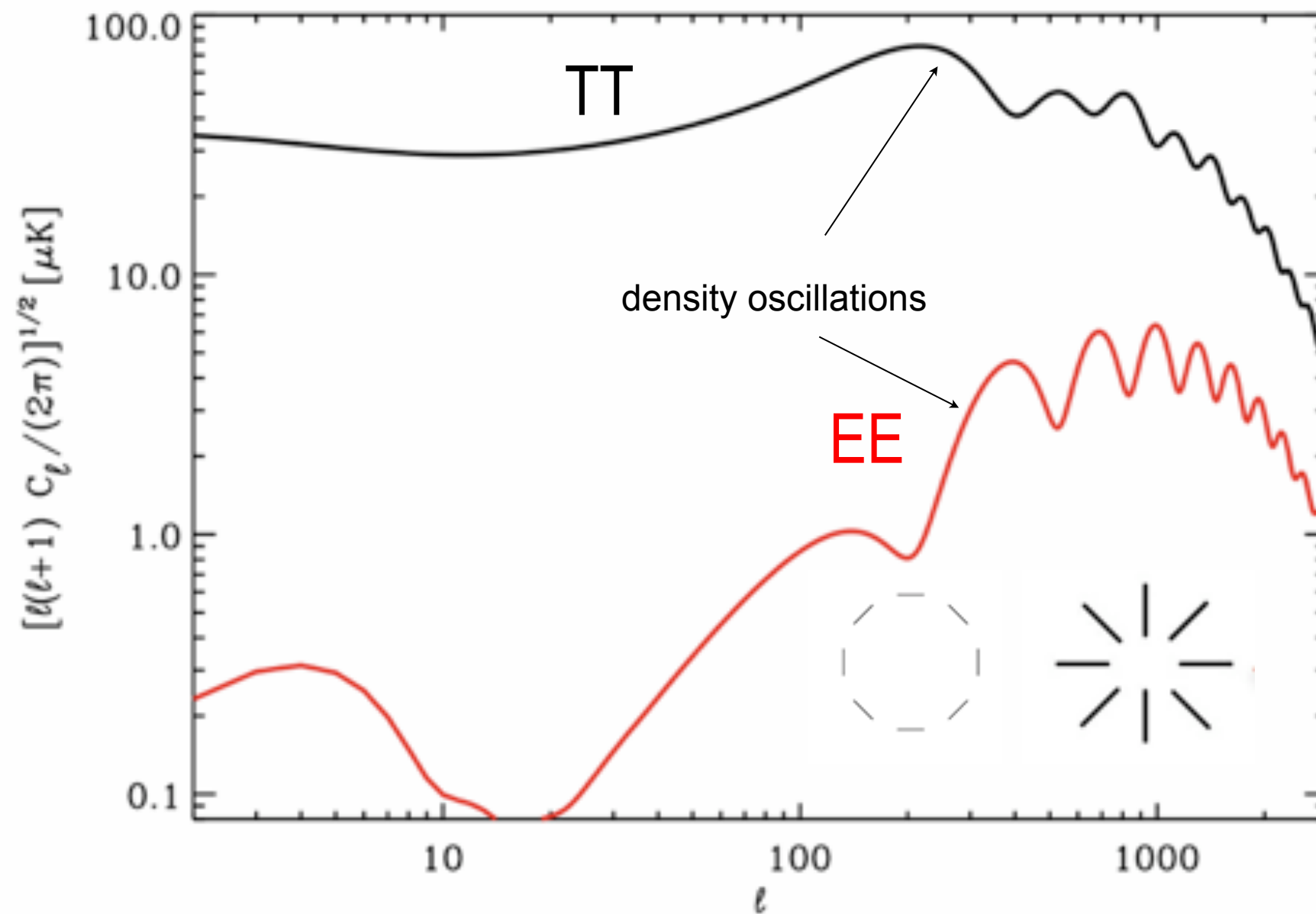
CMB polarization:
the next frontier for lensing & inflation

TT

density oscillations

EE

BB$_{IGW}$

Inflationary Gravitational wave oscillations

CMB polarization:
the next frontier for lensing & inflation

CMB polarization:
the next frontier for lensing & inflation

lensing of EE to BB
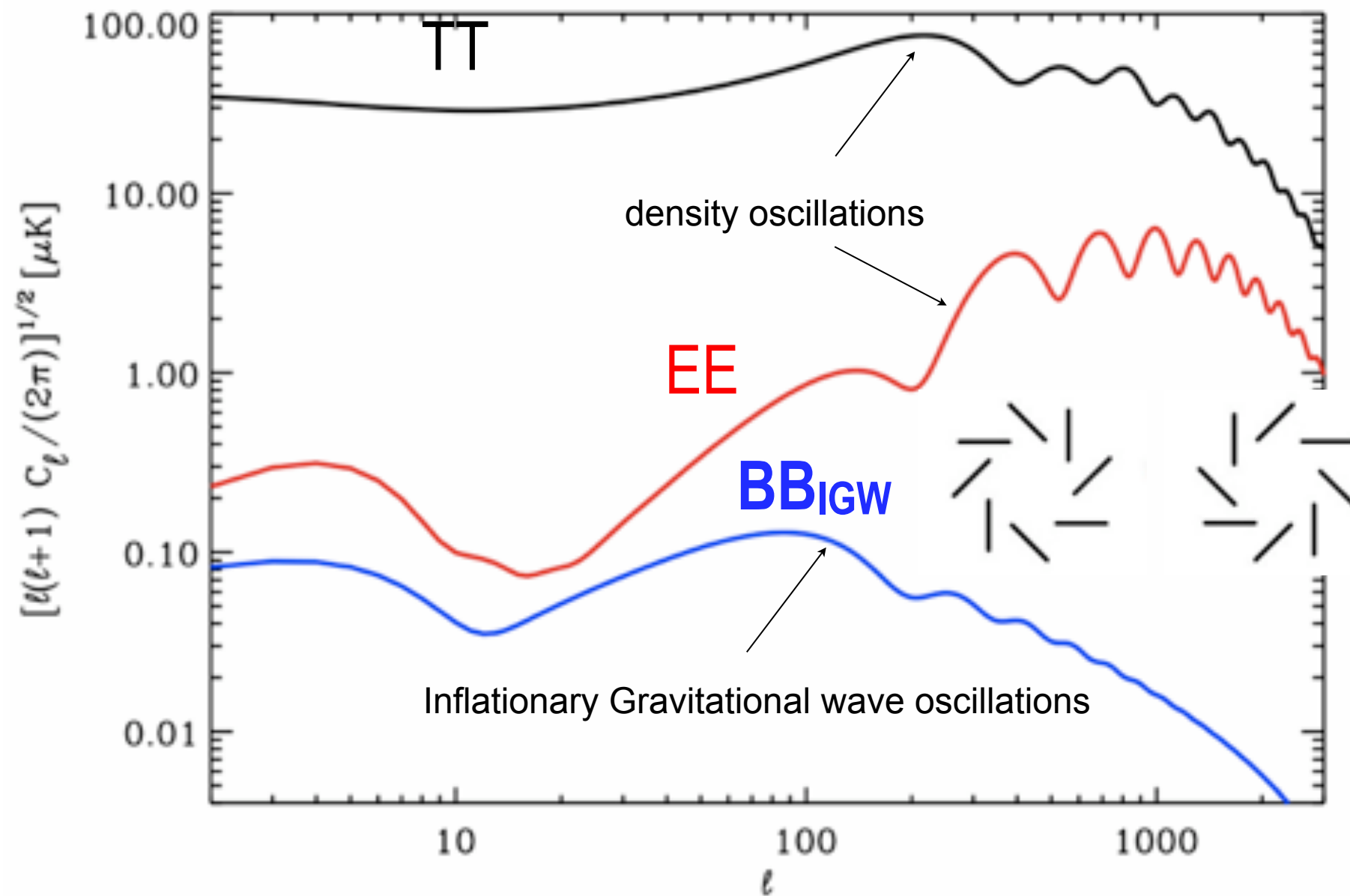
CMB polarization:
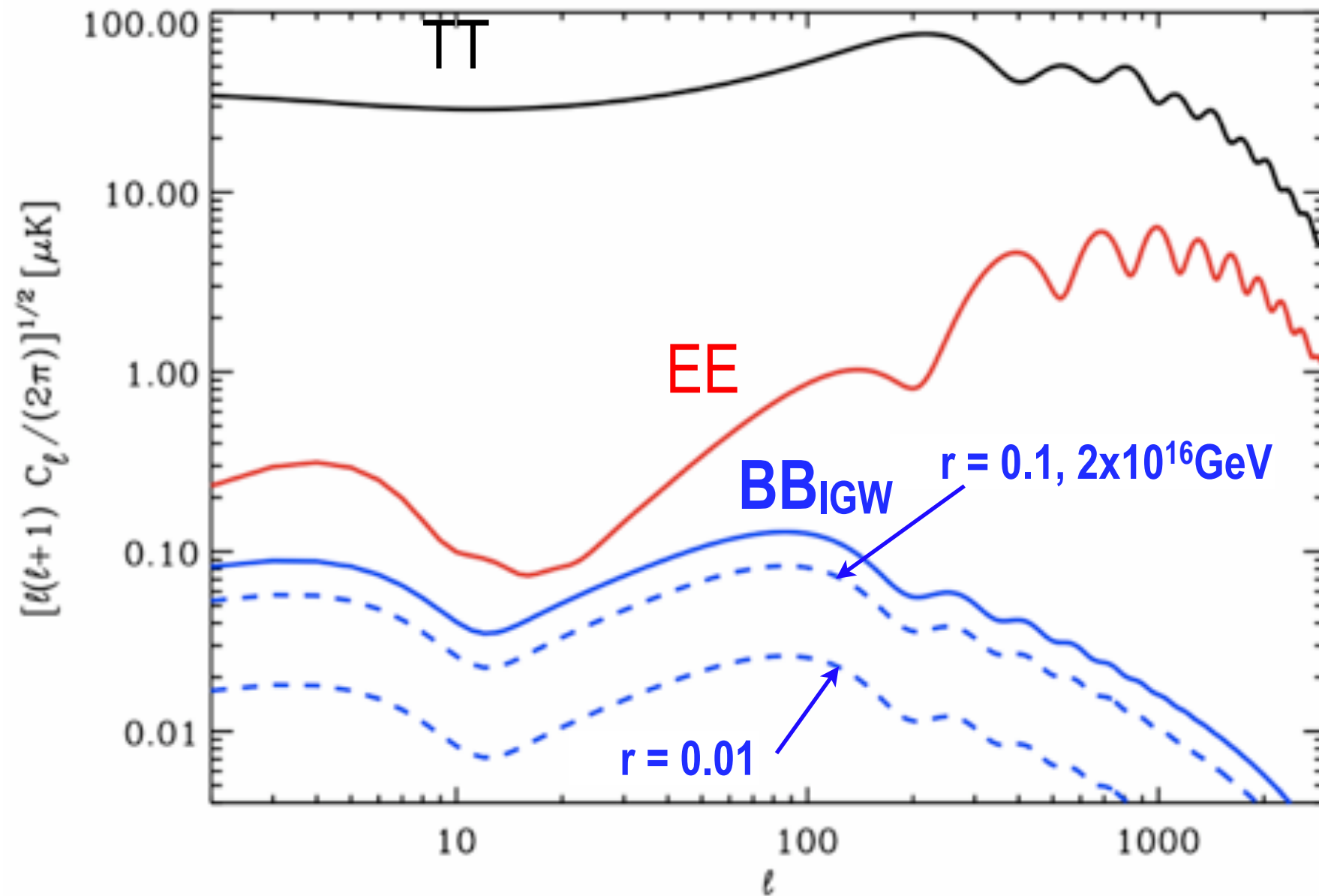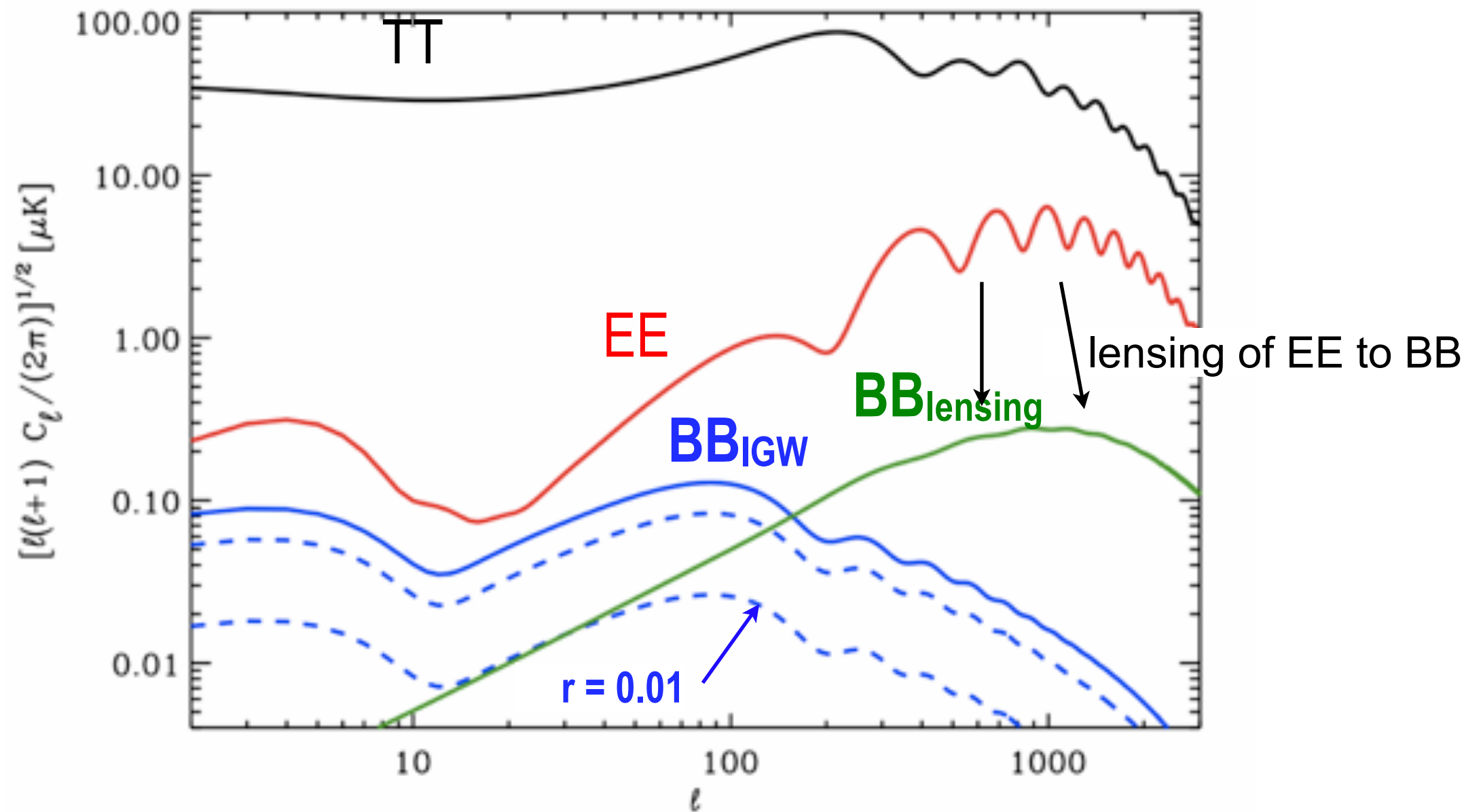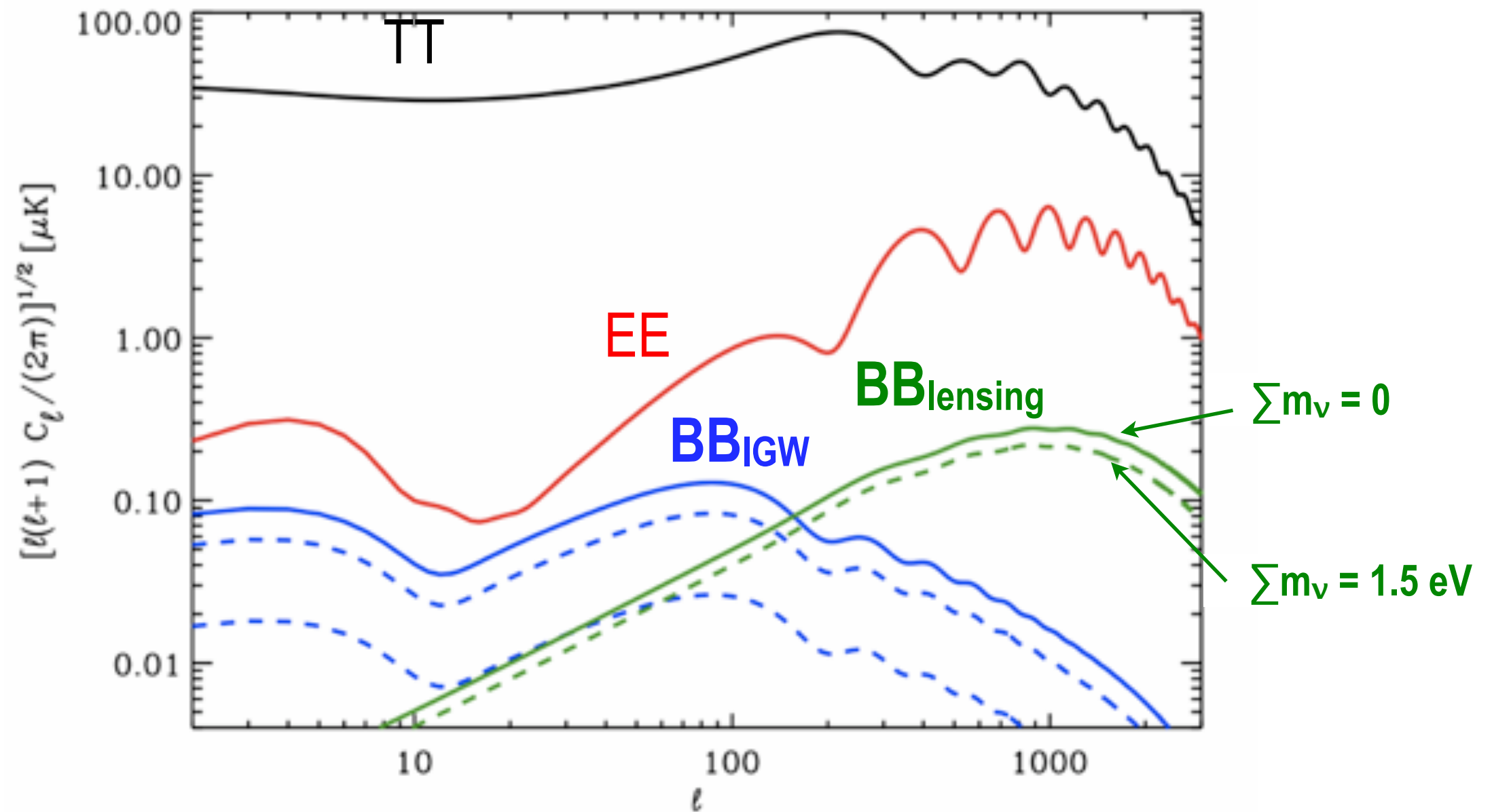## *the next frontier for lensing & inflation*

CMB polarization:
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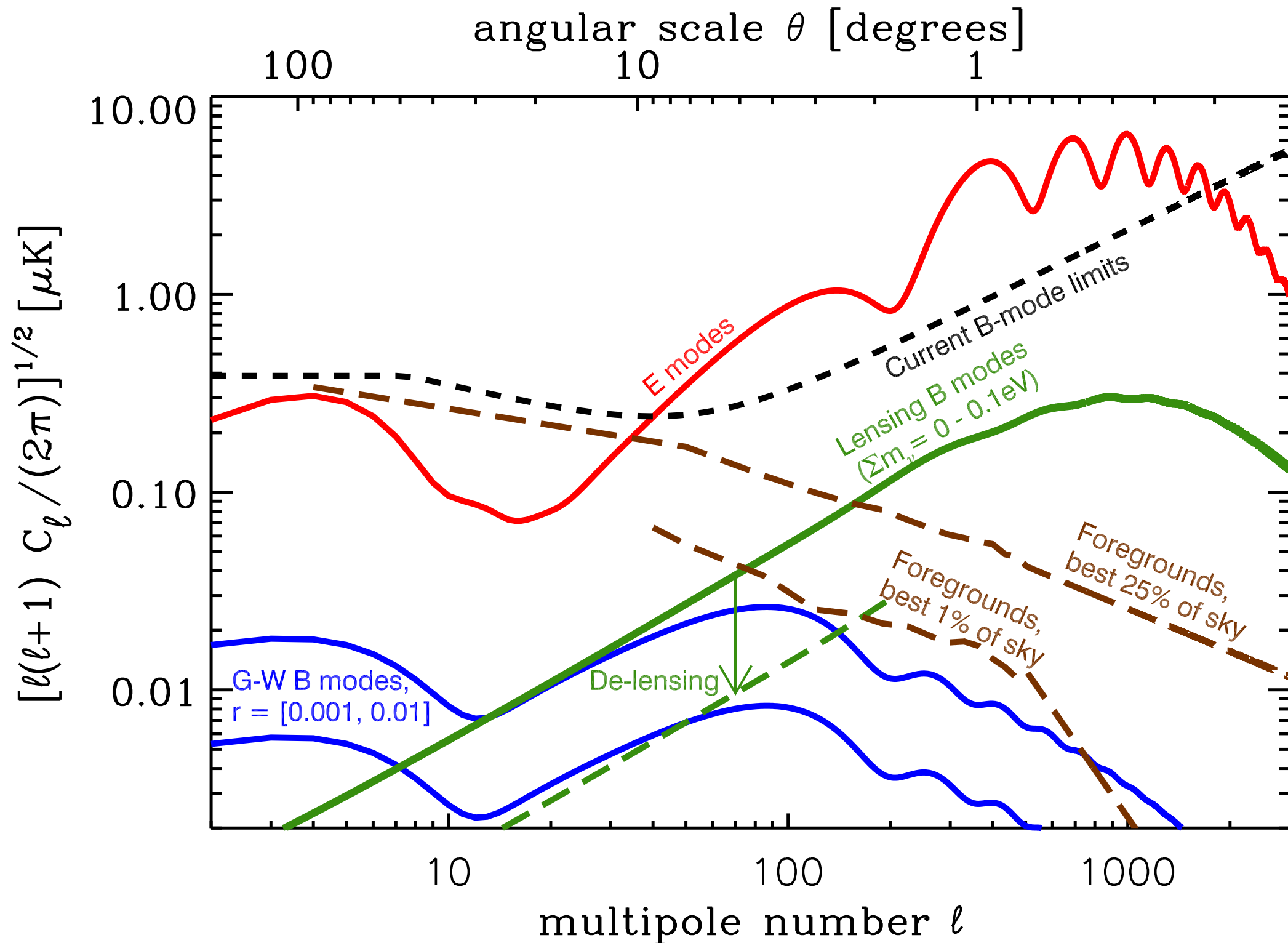
Figure from CF5 inflation doc: note expanded scale with 0.001 < r < 0.01

Status of B-mode experiments

Barkats et al., arXiv:1310.1422

EE: > 2σ detections

BB: 95% C.L. upper limits

— 100 nK !

BICEP

QUAD

QUAD
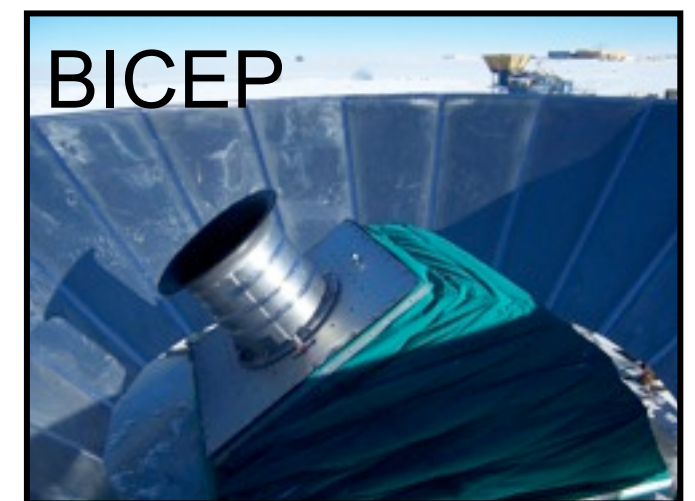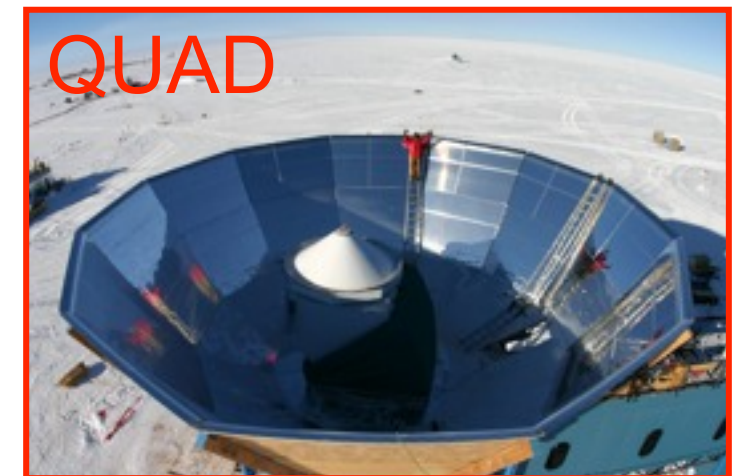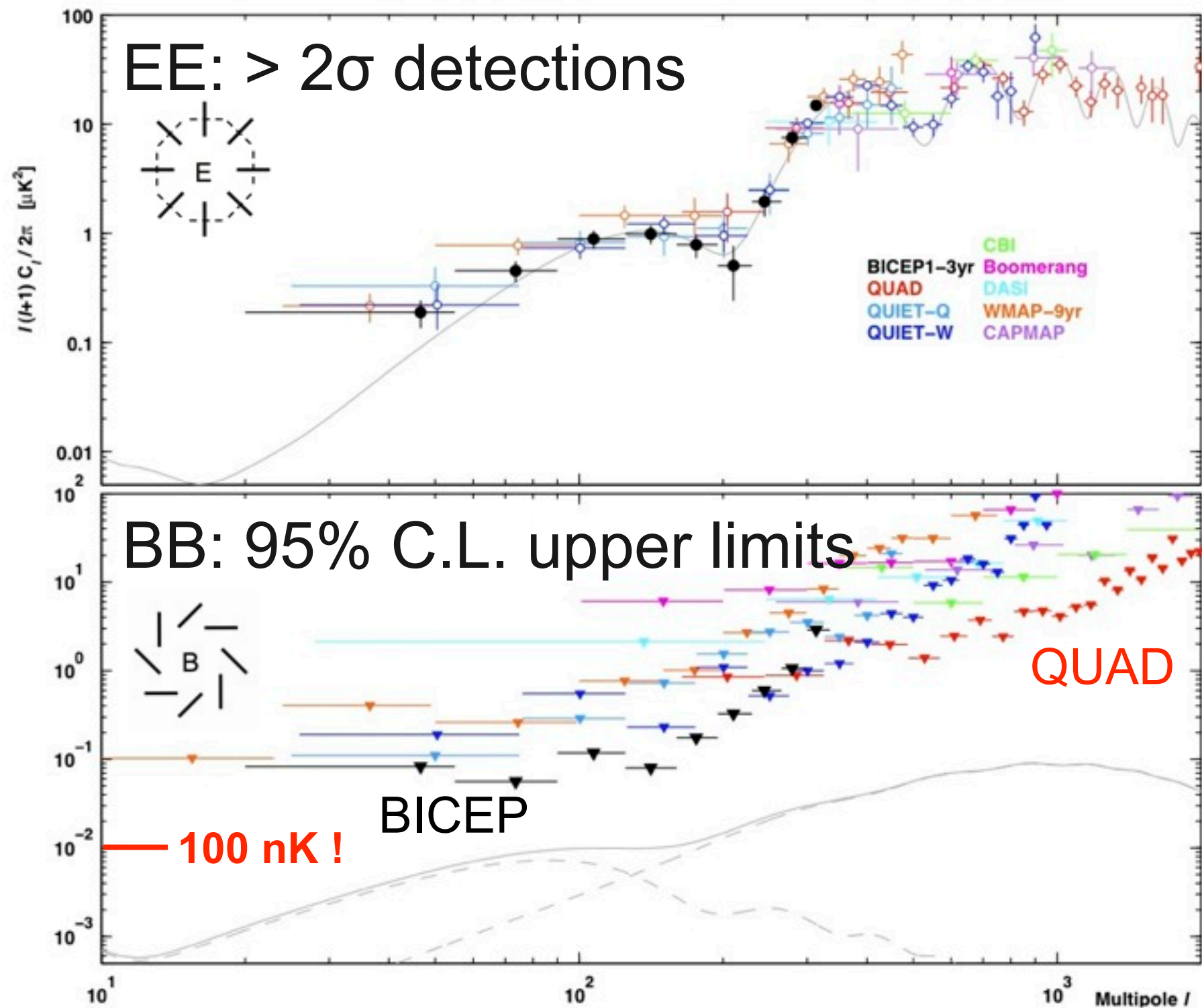
BICEP

SPTpol Detection of lensing B-modes
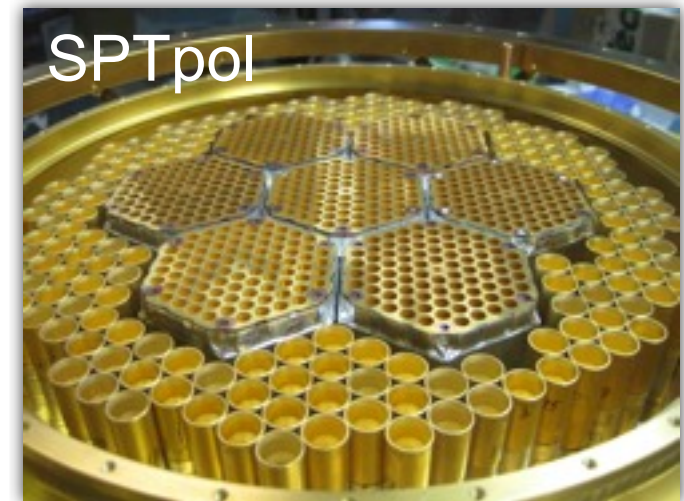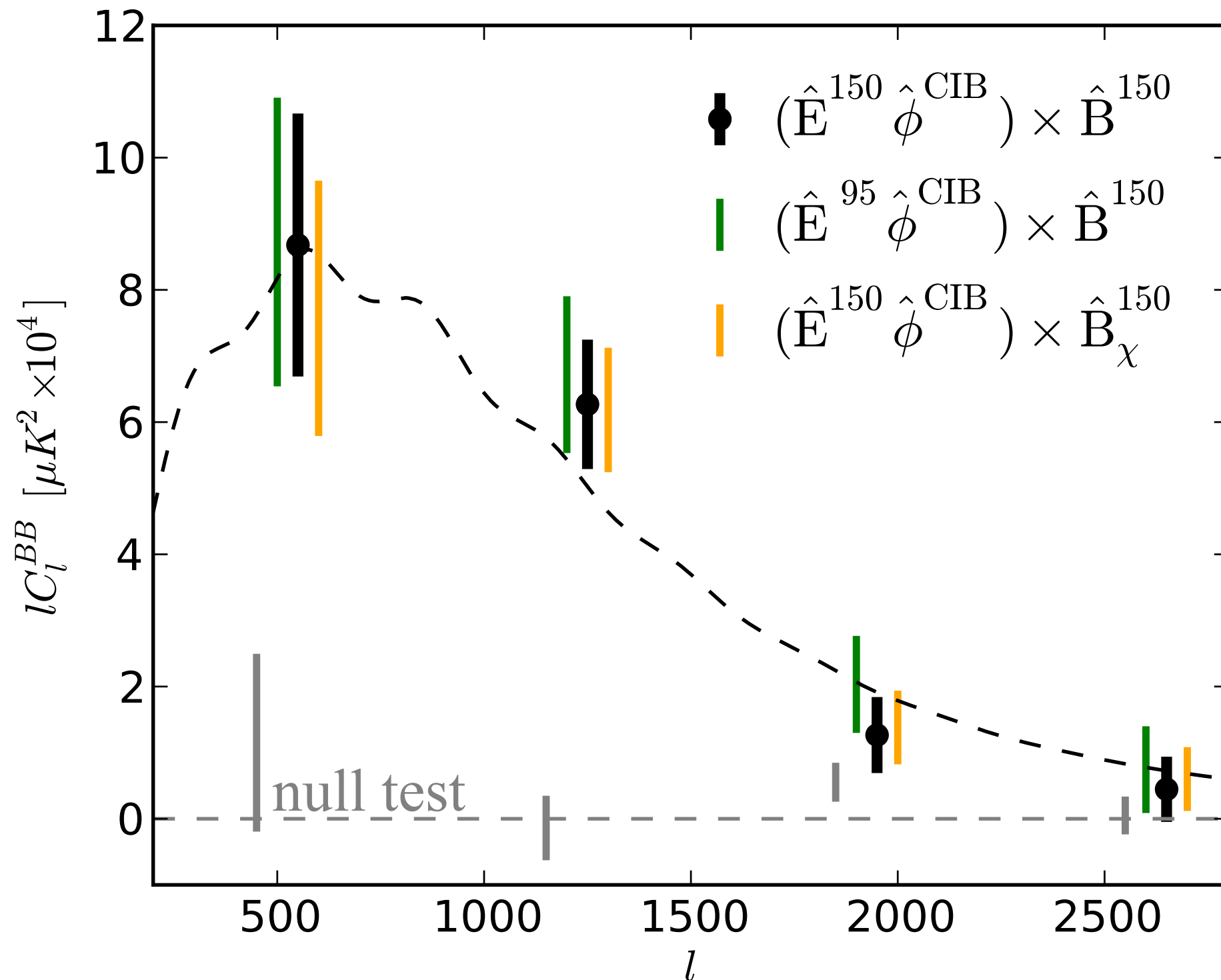
CMB timeline

- 2009: r < 0.7 (BICEP) Chiang et al, 0906.1181

- 2013: Stage II experiments detect lensing B-modes

- 2014: r ≲ 0.1 from Inflationary B-modes (BICEP 2) ?

- 2013-2016: Stage II experiments
  $\sigma(r) \sim 0.03$, $\sigma(N_{eff}) \sim 0.1$, $\sigma(\Sigma m_\nu) \sim 0.1 eV$

- 2016-2020: Stage III experiments
  $\sigma(r) \sim 0.01$, $\sigma(N_{eff}) \sim 0.06$, $\sigma(\Sigma m_\nu) \sim 0.06 eV$;

- 2020-2025: Stage IV experiment, **CMB−S4**
  $\sigma(r) = 0.001$, $\sigma(N_{eff}) = 0.020$, $\sigma(\Sigma m_\nu) = 16\ meV$

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  $\sigma(r) = 0.001$, $\sigma(N_{eff}) = 0.020$, $\sigma(\Sigma m_\nu) = 16$ meV

On an ambitious path forward and producing a steady flow of scientific results

CMB-S4 What it will deliver

Inflation projection for CMB-S4
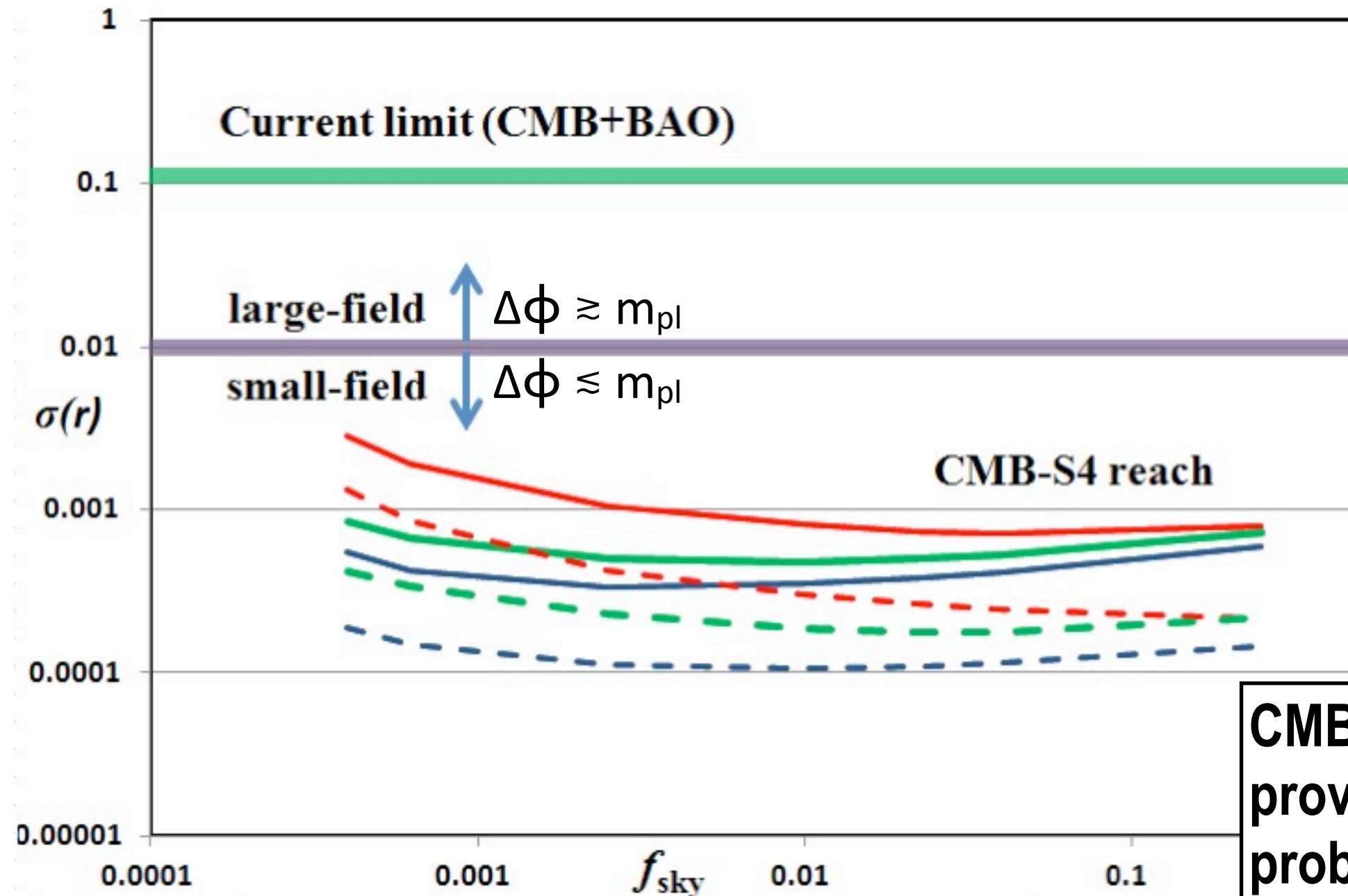
Current limit (CMB+BAO)

large-field $\Delta\phi \gtrsim m_{pl}$

small-field $\Delta\phi \lesssim m_{pl}$

$\sigma(r)$

CMB-S4 reach

f_{sky}

CMB polarization provides the only probe for r < 0.1

—— 3.5µK·√S 8'FWHM 10%FG —— 3.5µK·√S 1' FWHM 10%FG

—— 3.5µK·√S 1'FWHM   5%FG – – 1.1µK·√S 8'FWHM   5%FG

– · 1.1µK·√S 1'FWHM 10%FG – – 1.1µK·√S 1'FWHM   5%FG

CMB-S4 What it will deliver

Joint projections N_{eff} - Σm_ν

— Planck (no lensing) + DESI
— Stage IV CMB
— Stage IV CMB + DESI BAO
— Stage IV CMB (no lensing) + DESI
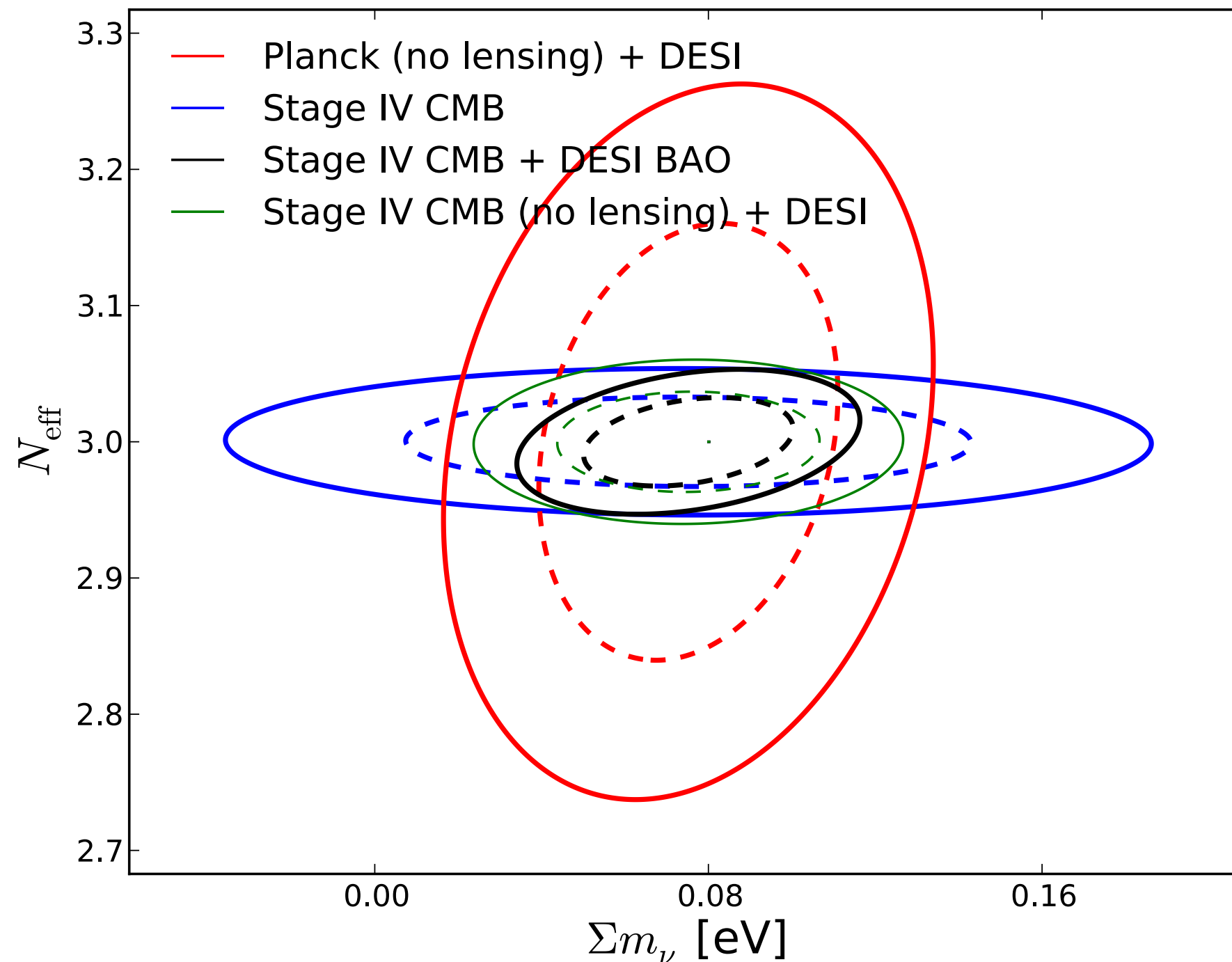
$\sigma(\Sigma m_\nu)$ = 16 meV *with two probes!*

$\sigma(N_{eff})$ = 0.020 *unique to CMB*

Our forecasters: J. Errard, P. McDonald, A. Slosar K. Wu, O. Zahn

CMB-S4 What it will take

- ## CMB-S4 Survey:

  - Maximum return on Inflation, Neutrino, and Dark Energy science requires an optimized survey which includes a range of resolution and sky coverage from deep to wide.

- ## Sensitivity of ~1 uK-arcmin over half the sky

- ## Experimental Configuration:

  - 200,000+ detectors on multiple platforms

  - spanning 40 - 240 GHz for foreground removal

  - ≲ 3 arcmin resolution required for CMB lensing & neutrino science,

  (*higher resolution leads to amazing and complementary dark energy constraints and gravity tests on large scales via the SZ effect*)

CMB-S4: How to do it

- Build on extensive experience from earlier generation CMB experience
  - People
  - Technology
  - Systematic Error Control
  - Analysis
- And increase throughput by over an order of magnitude

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<u>Technical challenge:</u>
is the scaling of the CMB detector arrays.

<u>Sociological evolution:</u>
the highly competitive CMB groups are working together.

CMB-S4: How to do it

- Exploit superb, established sites at Atacama Chile and South Pole
  - proven high and dry sites for sensitive CMB measurements
  - provides the required access to > 50% of the sky, including coverage of the optical survey fields

CMB S4 Large Area Survey Region

(overlap with LSST, MS-DESI, etc)
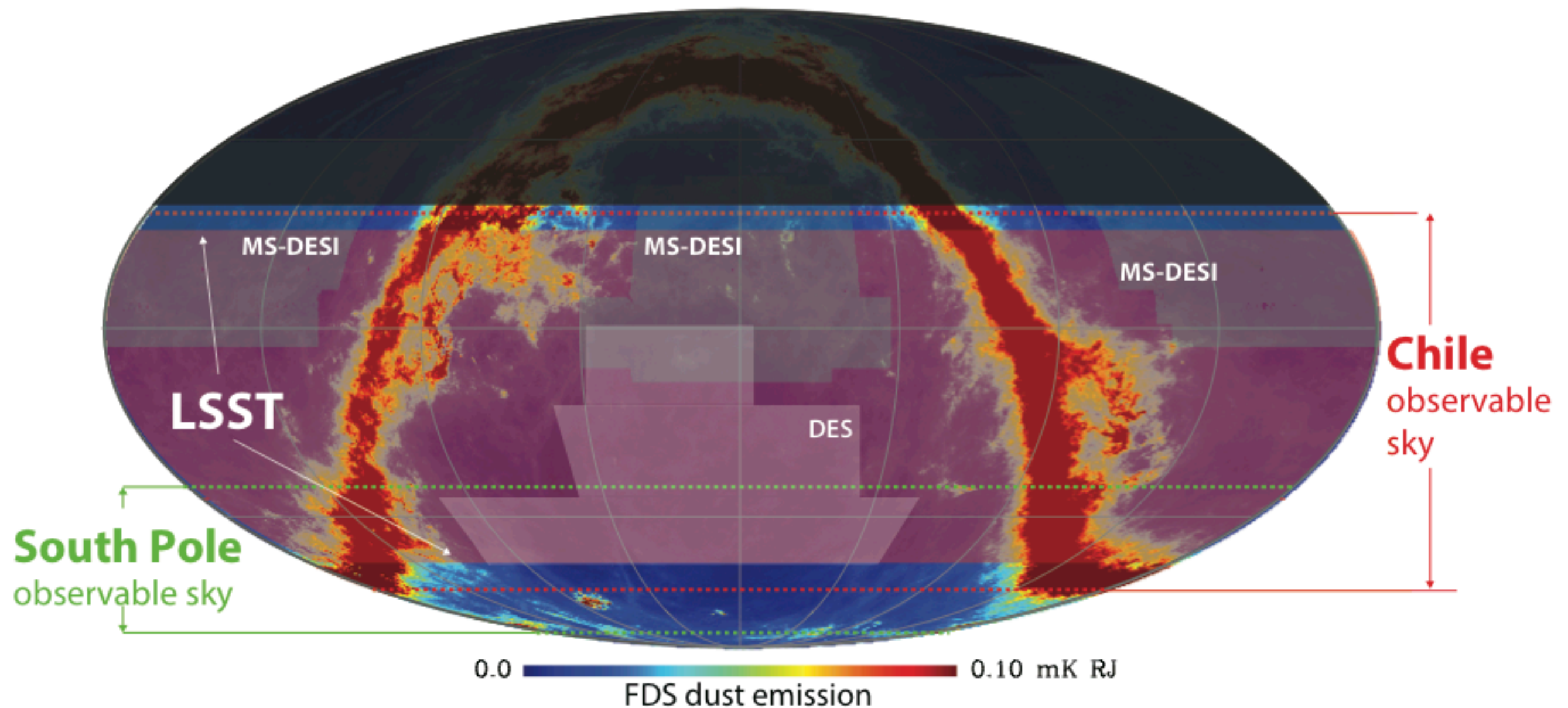
Build on investment by NSF in Chilean mid-Latitude Facilities and CMB experiments

to PolarBear

ACT

to ALMA
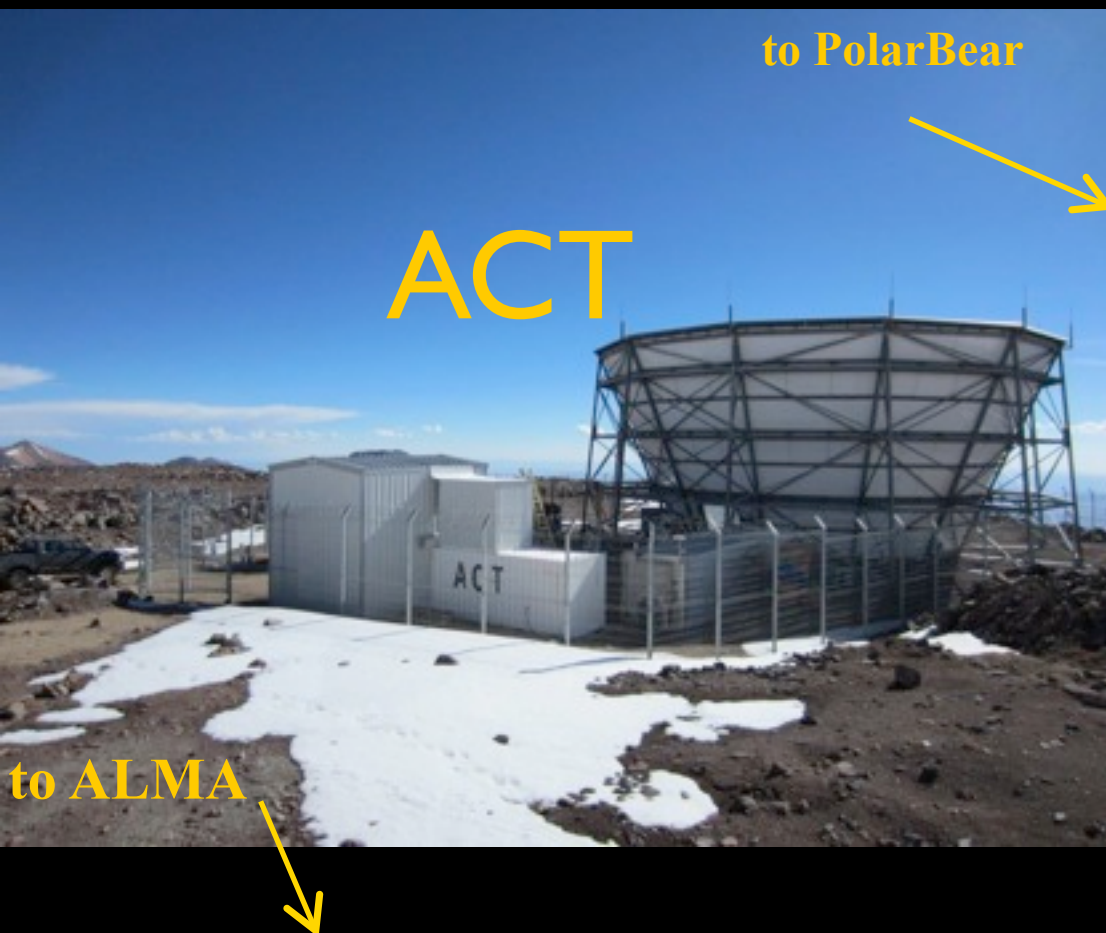
5200 meter (17,000 ft) site developed by the ACT team since 1998 provides access to over 50% of the sky

- The Atacama Cosmology Telescope
    - 6 meter aperture (1.4 arcmin at 150 GHz)
- Polarbear Telescope
    - 3.5 meter (3.5 arcmin at 150 GHz)

Polarbear

$15M+ in Telescopes and Logistics

- Power, internet, workspace, roads
- Machine shop and supplies
- Low altitude control room and housing
- Ties to Chilean contractors and suppliers
- Legal presence in Chile
- Established positive working relationship with CONICYT (Chilean NSF)

Build on investment by NSF in South Pole Facilities and CMB experiments

- Major NSF research station (not shown) with excellent logistical support
- CMB measurements since the 1980s;
  Martin A. Pomerantz observatory established in 1994.
- Exceptionally low atmospheric noise (sky-noise) due to dry and stable atmosphere.
- Access to ~4000 square degrees of low foreground sky (10%), which is observable year-round, 24 hrs/day

CMB-S4: How to do it

UC Berkeley prototype pixel

Micro-strip to inline filters

TES Detector

Broad-band Polarization Sensitive Antenna

5 mm

75mm wafers
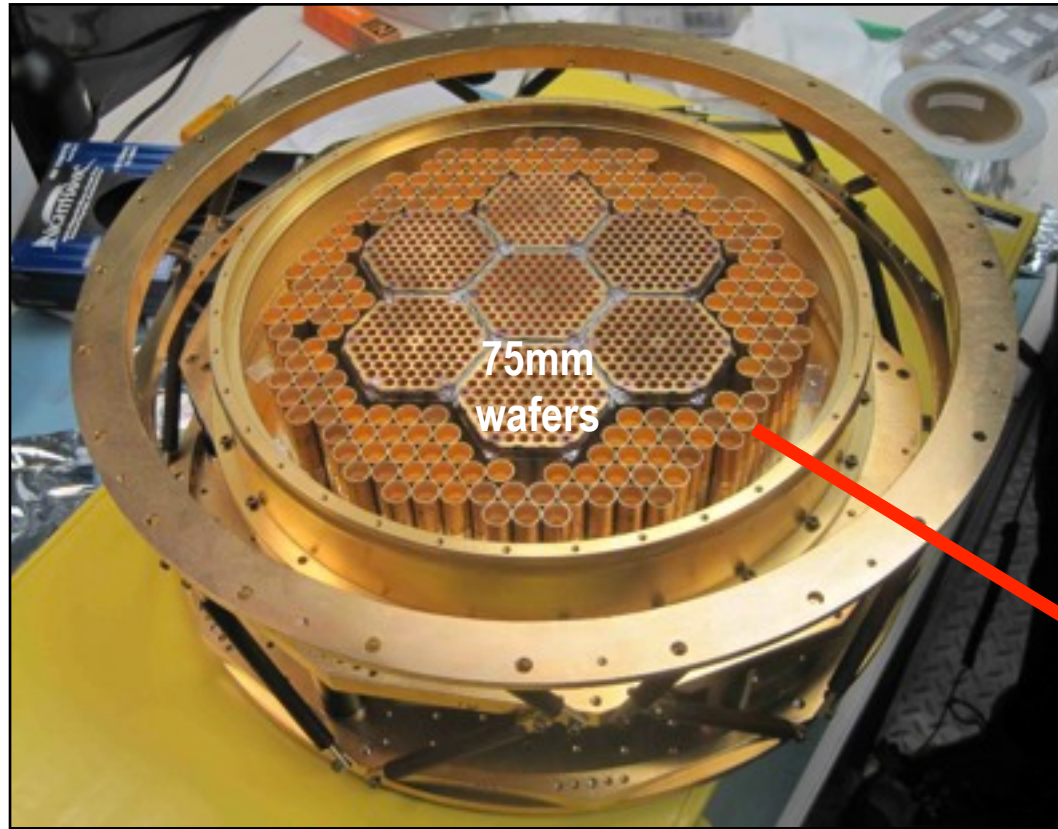
2012: SPTpol Stage II
1600 detectors (ANL/NIST)

ANL, LBNL, SLAC, Polarbear and SPT teams working on Stage II to Stage III detector advance based on UCB 3-band, dual polarization pixel; ACT team also working on multichroic pixel.

‣ **Optimized** *with background limited noise and high throughput*

‣ **Uniform** *properties over 150-mm diameter wafers*

‣ **Consistent** *fabrication from batch-to-batch*

150mm wafers

2016: SPT-3G Stage III 4x larger area
15,234 detectors at T = 250mK

CMB-S4: How to do it

Stage-3
~15,200 detectors
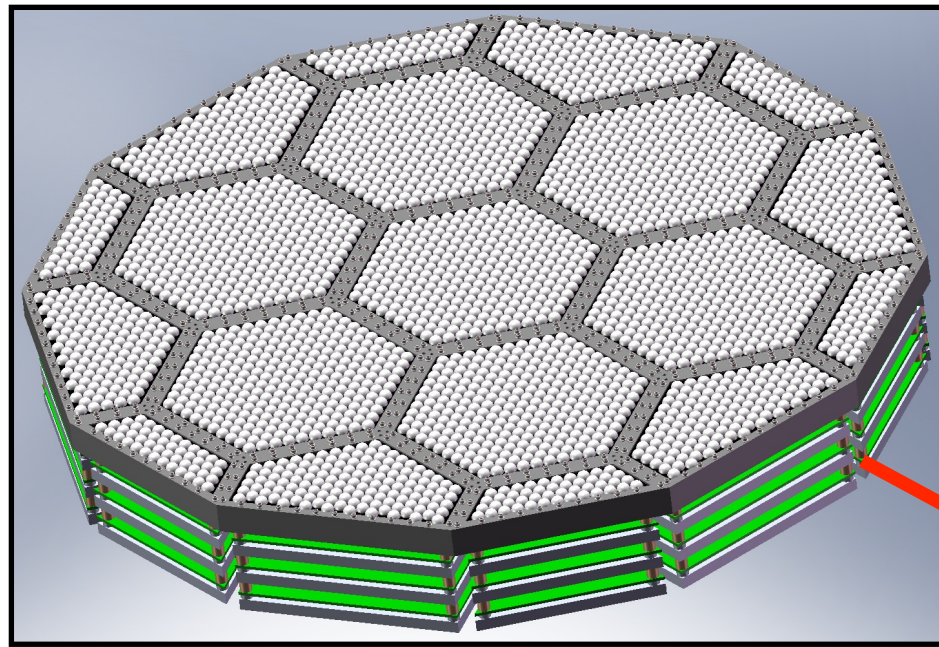
Stage-4

2020+: CMB-S4

200,000+ detectors
multiple telescopes
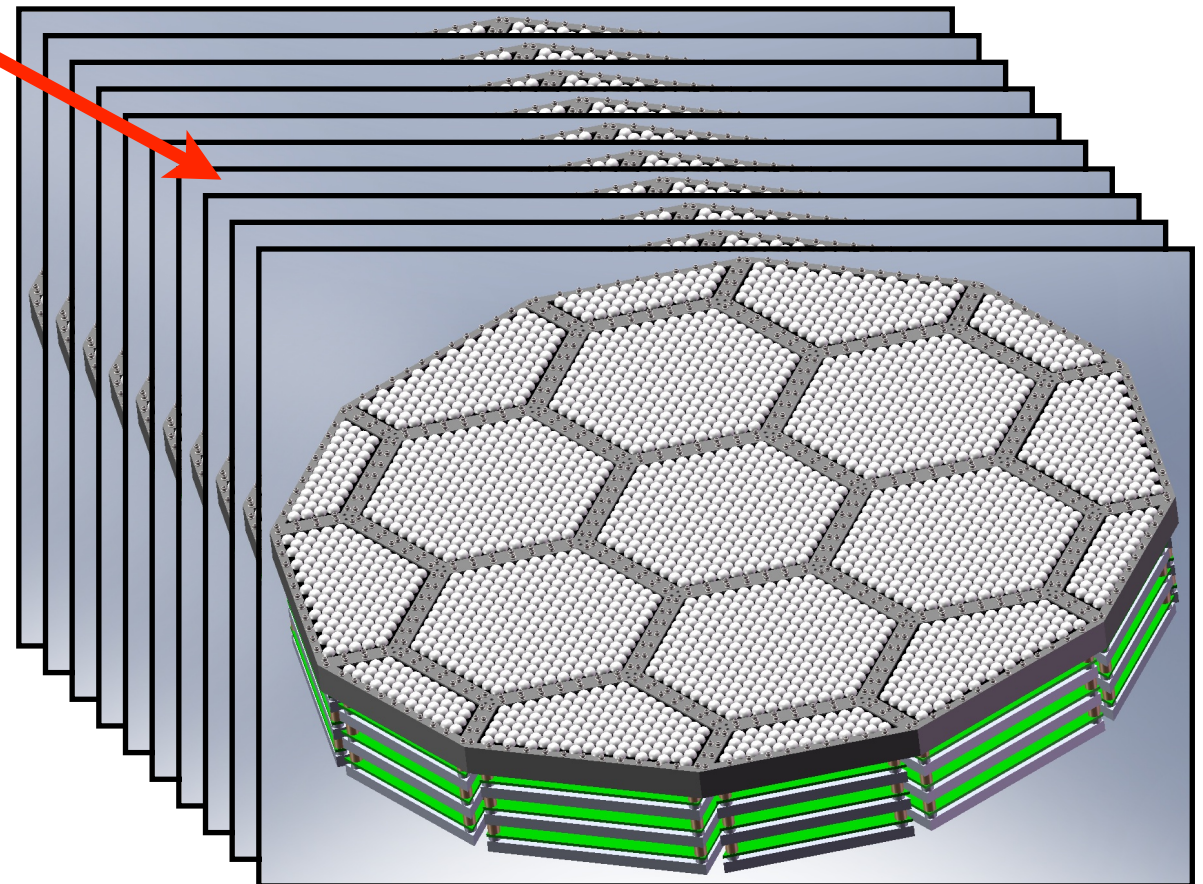
Detector sensitivity has been limited by photon "shot" noise for last ~15 years; further improvements are made only by making **more detectors.**

CMB-S4: How to do it

→ CMB-S4 requirements exceed capabilities of the traditional University-based CMB groups

- Increased production scope and reliability
  - 200,000+ detectors requires production of approximately 150 silicon 6" detector arrays
- Multiplexed TES Readout
- Large Cryogenic Optics
- Computing Infrastructure and Analysis tools
  - ~10,000 x *Planck* data size (~ 6 TB/day)
- Project Organization/Management

→ requires DOE National lab and HEP community working with the University-based CMB groups

Argonne National Laboratory

- Investment in robust, large scale detector fabrication.
- Involvement with SPTpol Stage II experiment (provided 90 GHz channel).
- Involvement in SPT-3G Stage III, providing detectors.

Fermilab

- Detector testing, SiDet for module assembly, and radiometer cryostat design, testing and integration.
- Experience with QUIET detector module testing and assembly.

CMB-S4

Berkeley Lab

- Investment in multiplexer readout.
- CMB heritage and connections with UCB detector development.
- High performance computing/ massively parallel data analysis.
- Involvement in Polarbear and SPT (Stage II & III).

SLAC National Accelerator Laboratory

- Investment in developing large aperture cryogenic optics.
- Investing in robust, large scale detector fabrication
- Investment in SQuIDs.
- Involvement in BICEP / KECK, SPT and ACT Stage III, providing detectors.

FY14	FY15	FY16	FY17	FY18	FY19	FY20	FY21 - FY24+

Establish collaboration &
management structure
◆ CD0

Detector & Readout R&D at
Universities and DOE Labs

DOE LAB Detector Fabrication & Readout

Optimize throughput of existing CMB telescopes
New focal planes and cameras

Design(s) of new telescopes

Build, deploy, commission new telescopes

NERSC Computing
(NERSC 8 deployed)   (NERSC 9 deployed)

transition from Stage III to coherent Stage IV CMB program

Full CMB-S4 Operations

NOTIONAL BUDGET		FY15	FY16	FY17	FY18	FY19	FY20	
DOE ANL/LBNL/SLAC Detector	project capital $*	0.5M	1M	1.5M	1.5M	1.5M	1.5M	
	FTE†	5	7	10	14	16	16	
DOE (LBNL/NERSC) Computing	project capital $	-	-	-	-	-	0.5M	
	FTE	-	1	2	2	3	3	
DOE Receiver test facilities, hardware & electronics	project capital $	-	3M	4.5M	7.5M	7.5M	7.5M	
	FTE	4	8	10	12	12	12	
NSF University CMB Dev, Test, Ops and Analysis	Σ $	(7M) current	7M	7M	7M	8M	9M	
new telescopes (NSF) site/deploy (NSF/DOE)	project capital $	-	2M	7M	7M	7M	7M	
NSF & DOE new telescope operations	$	-	0.5M	2M	3M	4M	5M	
DOE Lab & Univ Analysis (converts)	FTE	6	10	16	24	30	30	

*2013 dollars †DOE Particle Physicist FTE

NOTIONAL BUDGET		FY15	FY16	FY17	FY18	FY19	FY20	Capital $+FTEyr
DOE ANL/LBNL/SLAC Detector	project capital $	0.5M	1M	1.5M	1.5M	1.5M	1.5M	7.5M
	FTE	5	7	10	14	16	16	68 yr
DOE (LBNL/NERSC) Computing	project capital $	-	-	-	-	-	0.5M	0.5M
	FTE	-	1	2	2	3	3	11 yr
DOE Receiver test facilities, hardware & electronics	project capital $	-	3M	4.5M	7.5M	7.5M	7.5M	31M
	FTE	4	8	10	12	12	12	58 yr
NSF University CMB Dev, Test, Ops and Analysis	Σ $	(7M) current	7M	7M	7M	8M	9M	
†new telescopes (NSF) site/deploy (NSF/DOE)	project capital $	-	2M	7M	7M	7M	7M	30M
NSF & DOE new telescope operations	$	-	0.5M	2M	3M	4M	5M	
DOE Lab & Univ Analysis (converts)	FTE	6	10	16	24	30	30	

†Roughly 2:1 ratio of cost of telescopes to costs of site prep and deployment

Total Project Capital: $69M and 137 FTE · yr (not including 25% contingency)

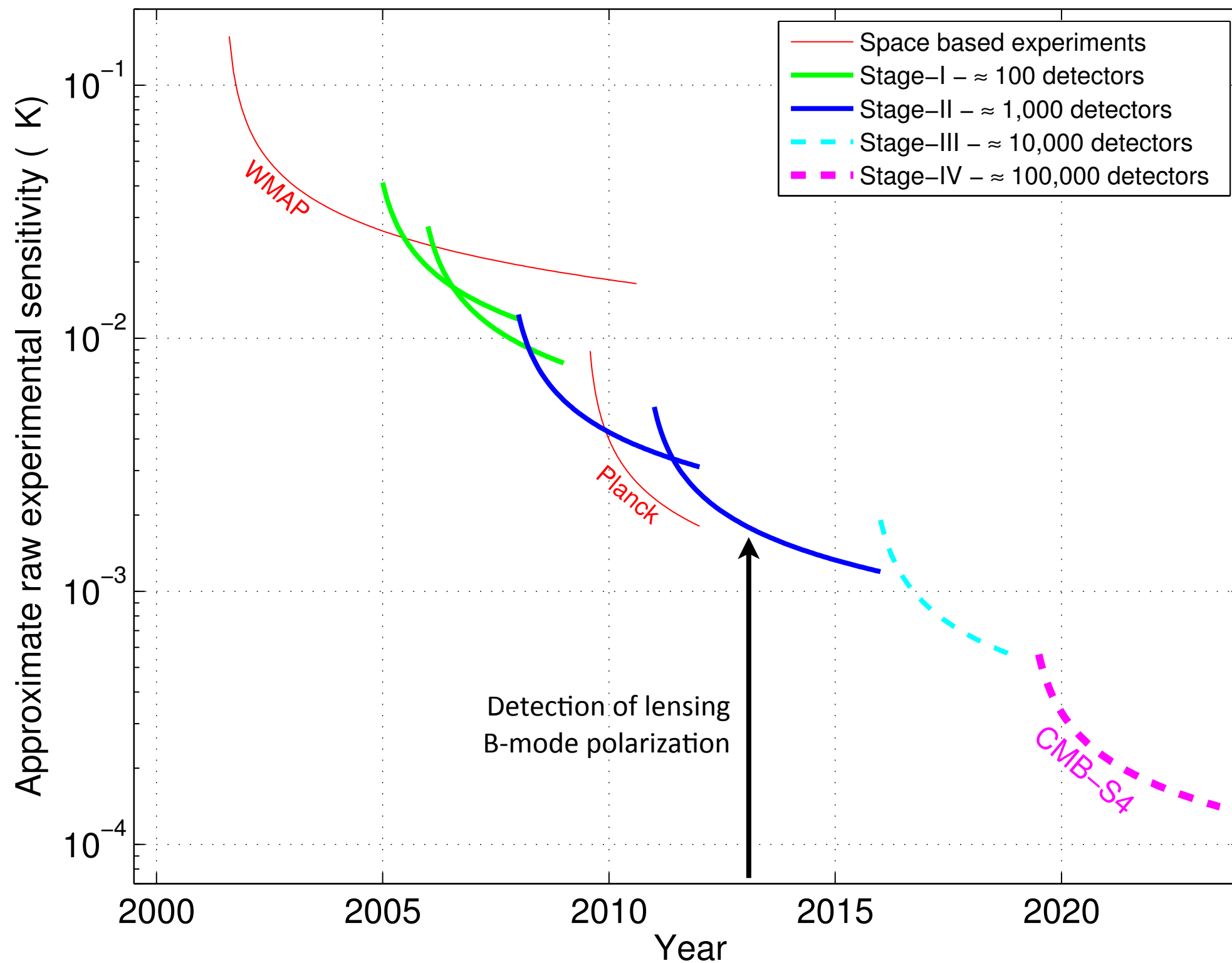| NOTIONAL BUDGET		FY15	FY16	FY17	FY18	FY19	FY20	Ops/yr Analysis/yr
DOE ANL/LBNL/SLAC Detector	project capital $	0.5M	1M	1.5M	1.5M	1.5M	1.5M	
	FTE	5	7	10	14	16	16	
DOE (LBNL/NERSC) Computing	project capital $	-	-	-	-	-	0.5M	0.5M/ 3yr
	FTE	-	1	2	2	3	3	4 FTE
DOE Receiver test facilities, hardware & electronics	project capital $	-	3M	4.5M	7.5M	7.5M	7.5M	
	FTE	4	8	10	12	12	12	
NSF University CMB Dev, Test, Ops and Analysis	Σ $	(7M) current	7M	7M	7M	8M	9M	9M
new telescopes (NSF) site/deploy (NSF/DOE)	project capital $	-	2M	7M	7M	7M	7M	
NSF & DOE new telescope operations	$	-	0.5M	2M	3M	4M	5M	5M
DOE Lab & Univ Analysis (converts)	FTE	6	10	16	24	30	30	30+FTE
		2020+ yearly ops and analysis: $14.2M and 34+ FTE						

International competition / partners

- There is no competition at the scale of CMB-S4
  - European L-class mission (PRISM) was turned down
  - No NASA mission expected on this time scale

- International partners
  - We envision CMB-S4 as primarily a U.S. project
  - Current international partners contributing to the CMB teams, e.g.,
    - Cardiff with all
    - KEK, Japan with Polarbear
    - McGill U with Polarbear and SPT
    - CITA, Oxford, UBC with ACT
    - Chile is critical partner; ACT works with CONICYT (Chilean NSF)
    - plus many other international participants
  - We expect a lot of international interest if CMB-S4 goes forward.

What we hope P5 will endorse

1. CMB uniquely addresses fundamental and exciting HEP science.

2. DOE-HEP has critical role in current and future CMB experiments.

3. Continued NSF and DOE funding of the CMB groups is critical to advancing CMB science.

   - It is essential to include the expertise from established university CMB groups.

   - Best and most economical path to CMB-S4 is to build on existing CMB experiments & telescopes.

4. CMB-S4 technology is identified and significantly mature to push for large scale integration; CMB-S4 detector development could and should start ASAP.

5. CMB-S4 should exploit infrastructure investments in robust, large scale micro-fabrication at ANL and SLAC, and in detector development at LBNL. Two production facilities will be needed.

6. CMB-S4 program could be ready for project CD0 in 2015, with full deployment in 2020 and measurements continuing through 2024.

7. The CMB program and CMB-S4 is expected to continue to produce a steady flow of scientific results and new discoveries en route to achieving its primary goals.
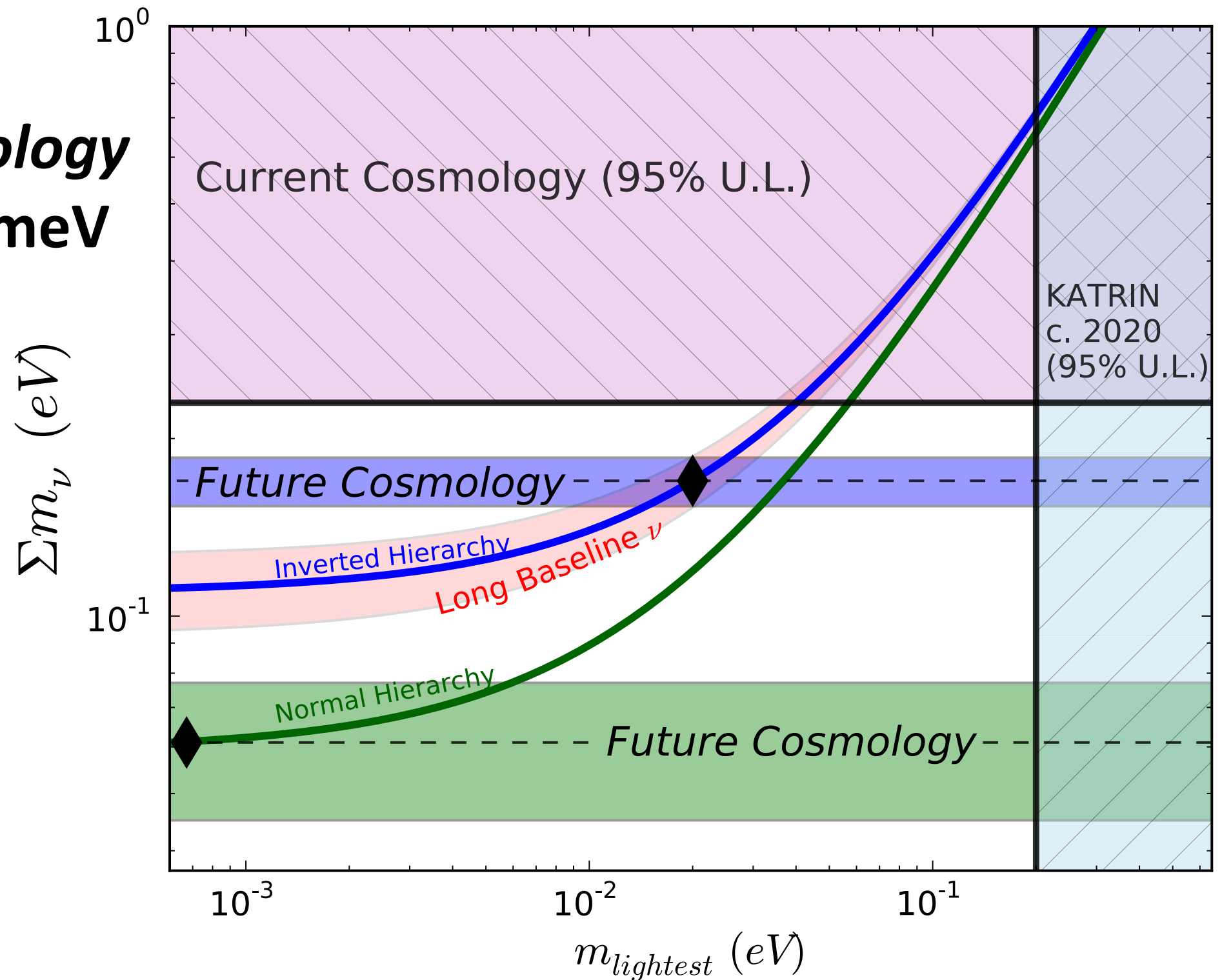
Extra slides

Combined Neutrino mass constraints

Future Cosmology
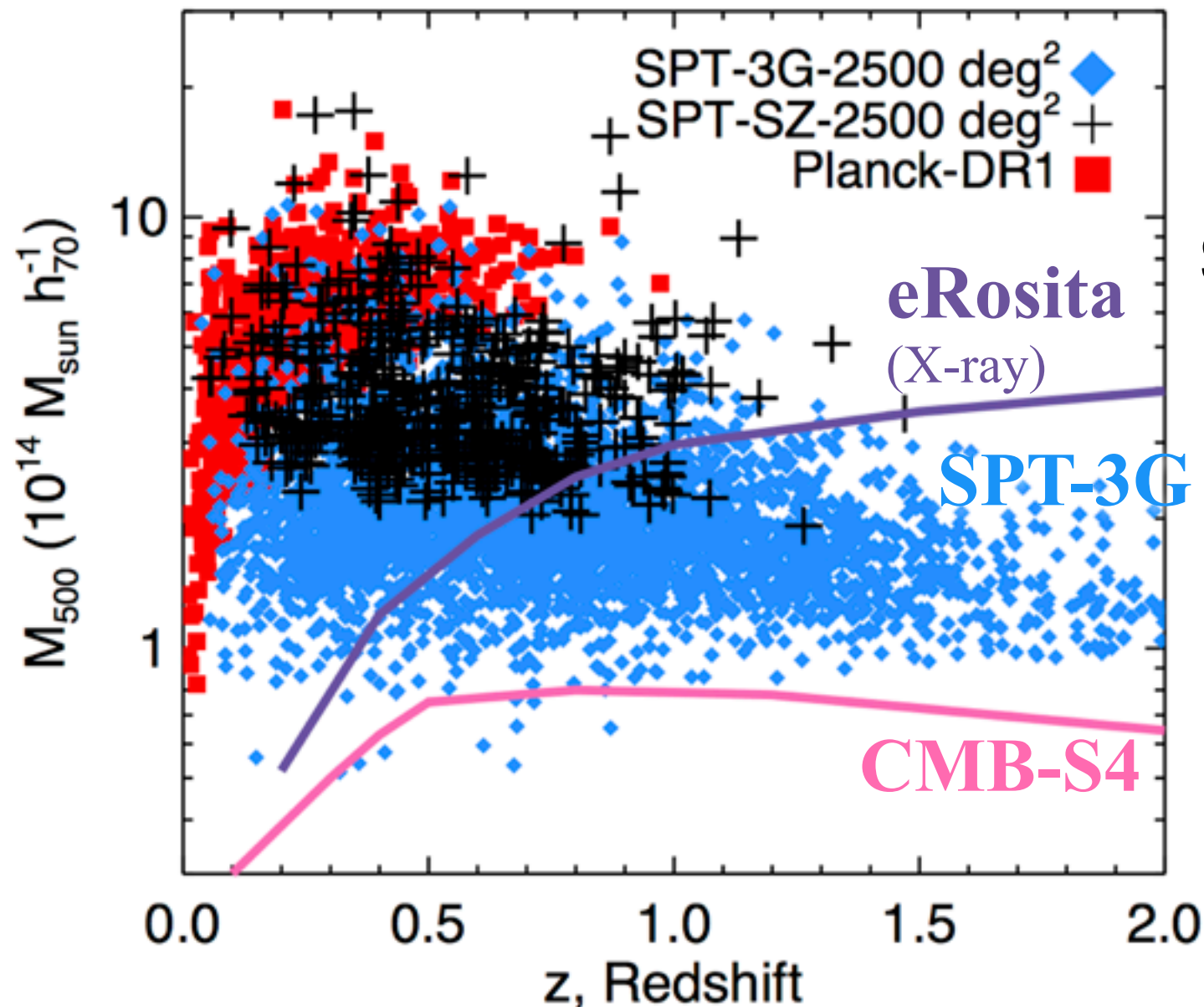**σ(Σm_ν) = 16 meV**

Current Cosmology (95% U.L.)

KATRIN c. 2020 (95% U.L.)

Σm_ν (eV)

Future Cosmology

Inverted Hierarchy

Long Baseline ν

Normal Hierarchy

Future Cosmology

$m_{lightest}$ (eV)

"use cosmology to tighten the noose" Boris Kayser

CMB Sunyaev-Zel'dovich Cluster Survey
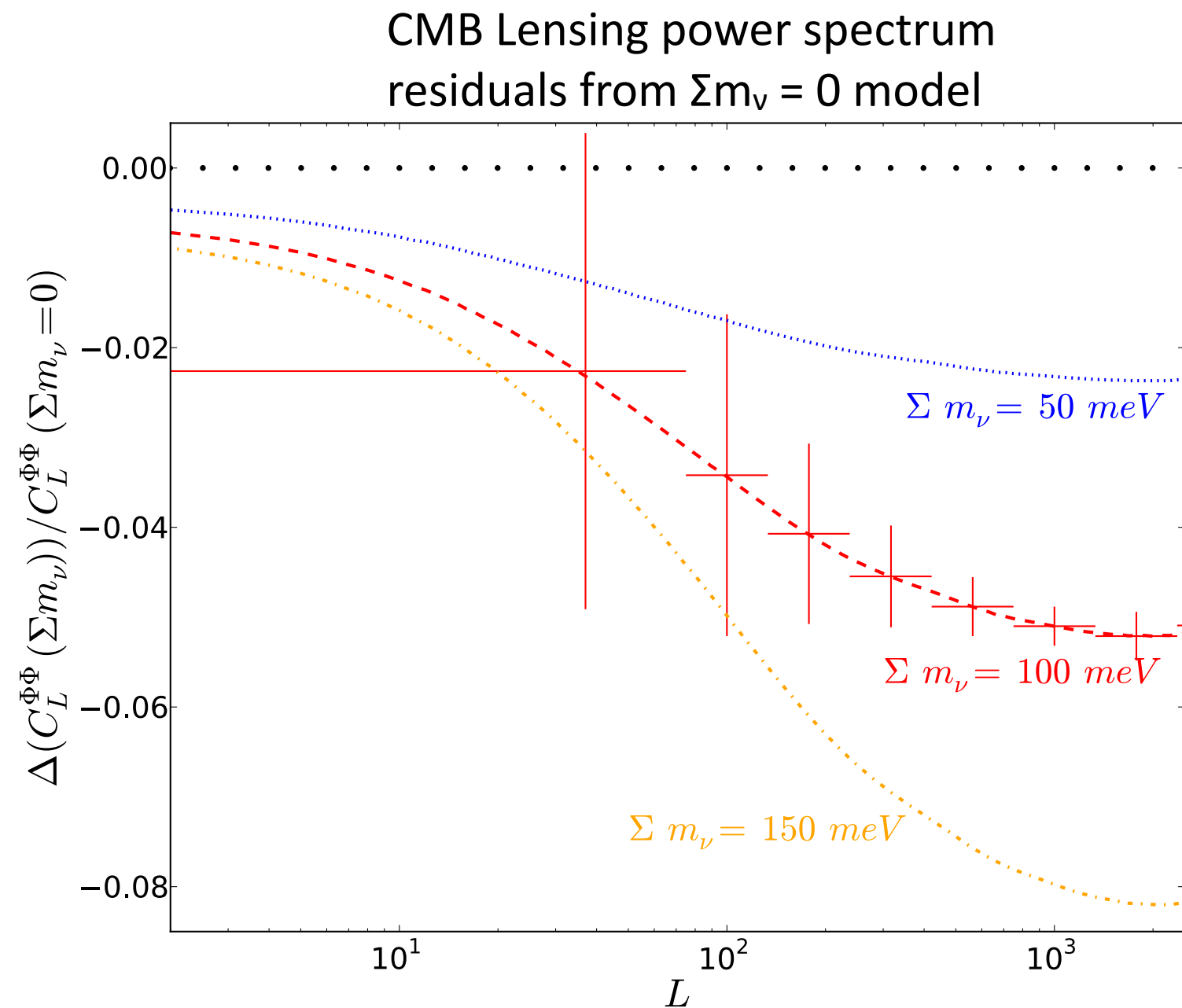
Cluster Mass vs Redshift from CMB SZ measurements

CMB measurements detect clusters through the "shadows" they make in the CMB, the Sunyaev-Zel'dovich (SZ) effect:

SPT-SZ/pol: N_{clust} **~ 1,000**

SPT-3G: N_{clust} **~ 10,000**

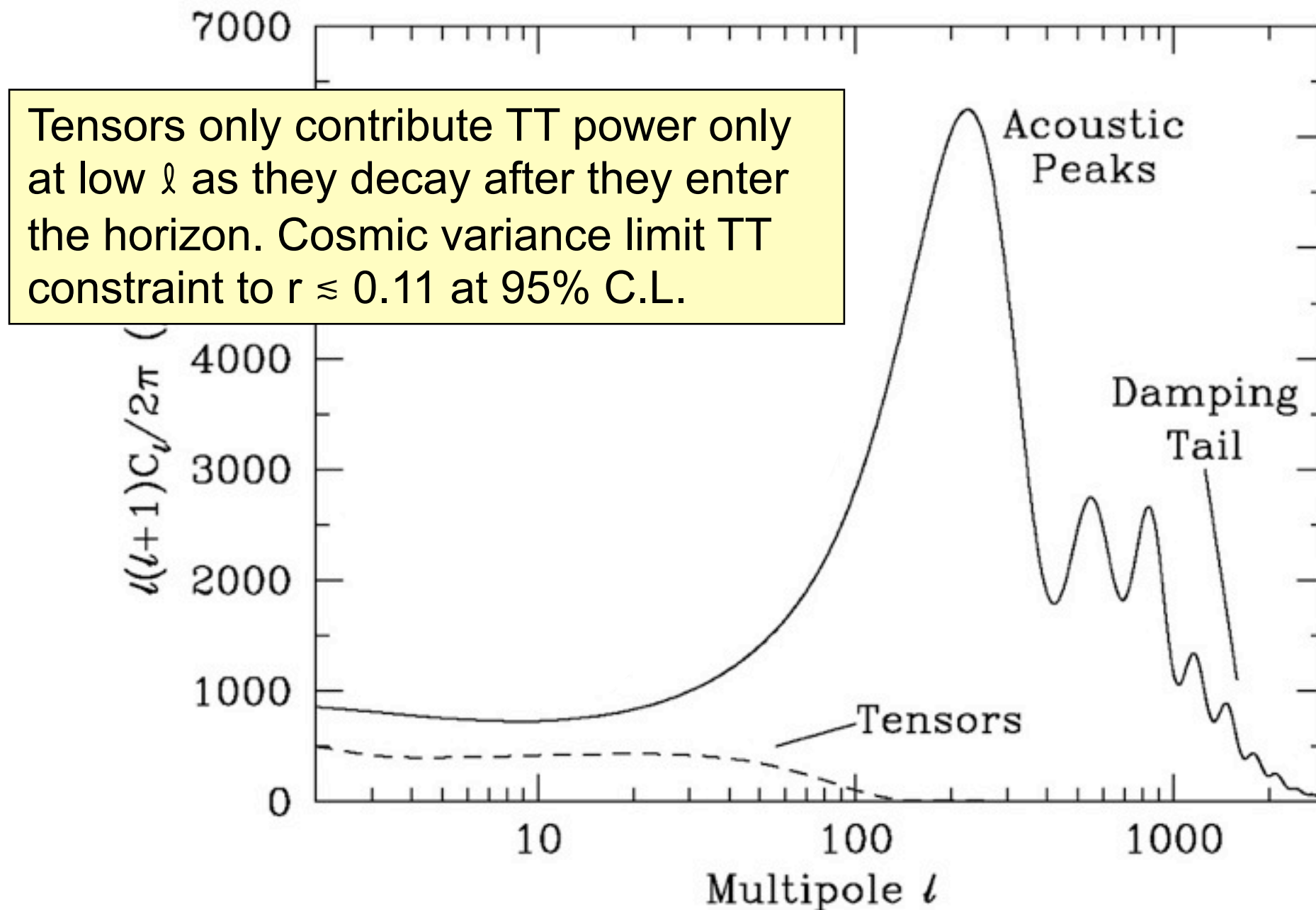**CMB-S4:** N_{clust} **~ 100,000+**

CMB lensing measured from individual clusters, can directly calibrate cluster mass:

SPT-3G: $\sigma(M)$ *~ 3%*

CMB-S4: $\sigma(M) < {\sim}0.1\%$

SPT-3G Collaboration

CMB-S4 Lensing Sensitivity Σm$_\nu$

CMB Lensing power spectrum
residuals from Σm$_\nu$ = 0 model

setting limit to tensor perturbations
i.e., primordial gravitational waves

Tensors only contribute TT power only at low ℓ as they decay after they enter the horizon. Cosmic variance limit TT constraint to $r \lesssim 0.11$ at 95% C.L.

$$r \equiv \frac{\text{Tensor (gravitational) perturbation amplitude}}{\text{Scalar (density) perturbation amplitude}}$$

$$V^{1/4} = 1.06 \times 10^{16} \text{GeV} \left(\frac{r}{0.01}\right)^{1/4}$$

plot taken from Ned Wright's web pages